# **Vector Spaces**

**Vector Spaces** 

Subspaces

Linear Independence and Spanning Sets

Bases

Dimension

Properties of Dimension

### **Determinants**

Determinant of a Matrix

Properties of Determinants of Matrices

## Eigenvalues

Eigenvalues and Eigenvectors

Properties of Eigenvalues and Eigenvectors

Similarity and Diagonalization

#### **Linear Transformations**

**Linear Transformations** 

Injective Linear Transformations

Surjective Linear Transformations

Invertible Linear Transformations

### Representations

**Vector Representations** 

Matrix Representations

Change of Basis

Orthonormal Diagonalization

#### **Preliminaries**

Complex Number Operations

Sets

We will have theorems in this section that connect nonsingular matrices with systems of equations, creating more opportunities for confusion. Convince yourself now of two observations, (1) we can decide nonsingularity for any square matrix, and (2) the determination of nonsingularity involves the solution set for a certain homogeneous system of equations.

Theorem		
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Proof		

Notice that it makes no sense to call a system of equations nonsingular (the term does not apply to a system of equations), nor does it make any sense to call a  $5 \times 7$  matrix singular (the matrix is not square).

# **Example S** A singular matrix, Archetype A

## Example S A singular matrix, Archetype A

Example HISAA shows that the coefficient matrix derived from Archetype A, specifically the 3 × 3 matrix,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

is a singular matrix since there are nontrivial solutions to the homogeneous system  $\mathcal{LS}(A,\mathbf{0})$  .

(in context)

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# Example NM A nonsingular matrix, Archetype B