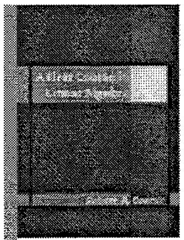

Nonsingular Matrices

In this section we specialize further and consider matrices with equal numbers of rows and columns, which when considered as coefficient matrices lead to systems with equal numbers of equations and variables. We will see in the second half of the course (Chapter D, Chapter E Chapter LT, Chapter R) that these matrices are especially important.

Subsection NM Nonsingular Matrices

Our theorems will now establish connections between systems of equations (homogeneous or otherwise), augmented matrices representing those systems, coefficient matrices, constant vectors, the reduced row-echelon form of matrices (augmented and coefficient) and solution sets. Be very careful in your reading, writing and speaking about systems of equations, matrices and sets of vectors. A system of equations is not a matrix, a matrix is not a solution set, and a solution set is not a system of equations. Now would be a great time to review the discussion about speaking and writing mathematics in Proof Technique L.

Definition SQM Square Matrix

A matrix with m rows and n columns is **square** if $m = n$. In this case, we say the matrix has **size** n. To emphasize the situation when a matrix is not square, we will call it **rectangular**. □

We can now present one of the central definitions of linear algebra.

Definition NM Nonsingular Matrix

Suppose A is a square matrix. Suppose further that the solution set to the homogeneous linear system of equations $\mathcal{LS}(A, \mathbf{0})$ is $\{\mathbf{0}\}$, in other words, the system has *only* the trivial solution. Then we say that A is a **nonsingular** matrix. Otherwise we say A is a **singular** matrix. □

We can investigate whether any square matrix is nonsingular or not, no matter if the matrix is derived somehow from a system of equations or if it is simply a matrix. The definition says that to perform this investigation we must construct a very specific system of equations (homogeneous, with the matrix as the coefficient matrix) and look at its solution set.

We will have theorems in this section that connect nonsingular matrices with systems of equations, creating more opportunities for confusion. Convince yourself now of two observations, (1) we can decide nonsingularity for any square matrix, and (2) the determination of nonsingularity involves the solution set for a certain homogeneous system of equations.

Notice that it makes no sense to call a system of equations nonsingular (the term does not apply to a system of equations), nor does it make any sense to call a 5×7 matrix singular (the matrix is not square).

Example S A singular matrix, Archetype A

Example HISAA shows that the coefficient matrix derived from Archetype A, specifically the 3×3 matrix,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

is a singular matrix since there are nontrivial solutions to the homogeneous system $\mathcal{LS}(A, \mathbf{0})$.

(in context)

./knowls/example.S.knowl

Example NM A nonsingular matrix, Archetype B