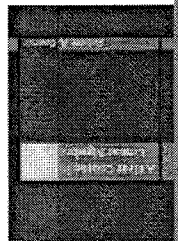


Nonsingular Matrices



A First Course in Linear Algebra
Preface
Dedication and Acknowledgements

Systems of Linear Equations

What is Linear Algebra?
Solving Systems of Linear Equations
Reduced Row-Echelon Form

Types of Solution Sets
Homogeneous Systems of Equations
Nonsingular Matrices

Vectors

Vector Operations

Linear Combinations
Spanning Sets

Linear Independence

Linear Dependence and Spans
Orthogonality

Matrices

Matrix Operations

Matrix Multiplication

Matrix Inverses and Systems of Linear Equations

Matrix Inverses and Nonsingular Matrices

Column and Row Spaces

Four Subsets

Subsection NM Nonsingular Matrices

Our theorems will now establish connections between systems of equations (homogeneous or otherwise), augmented matrices representing those systems, coefficient matrices, constant vectors, the reduced row-echelon form of matrices (augmented and coefficient) and solution sets. Be very careful in your reading, writing and speaking about systems of equations, matrices and sets of vectors. A system of equations is not a matrix, a matrix is not a solution set, and a solution set is not a system of equations. Now would be a great time to review the discussion about speaking and writing mathematics in Proof Technique I.

Definition SQM Square Matrix

A matrix with m rows and n columns is **square** if $m = n$. In this case, we say the matrix has **size** n . To emphasize the situation when a matrix is not square, we will call it **rectangular**. $\square$

We can now present one of the central definitions of linear algebra.

Definition NM Nonsingular Matrix

Suppose A is a square matrix. Suppose further that the solution set to the homogeneous linear system of equations $LS(A, 0)$ is $\{0\}$, in other words, the system has *only* the trivial solution. Then we say that A is a **nonsingular** matrix. Otherwise we say A is a **singular** matrix. $\square$

We can investigate whether any square matrix is nonsingular or not, no matter if the matrix is derived somehow from a system of equations or if it is simply a matrix. The definition says that to perform this investigation we must construct a very specific system of equations (homogeneous, with the matrix as the coefficient matrix) and look at its solution set.

Vector Spaces	Vector Spaces	Subspaces	Linear Independence and Spanning Sets	Bases	Dimension	Properties of Dimension	Determinants	Determinant of a Matrix	Properties of Determinants of Matrices	Eigenvalues	Eigenvalues and Eigenvectors	Properties of Eigenvalues and Eigenvectors	Similarity and Diagonalization	Linear Transformations	Linear Transformations	Injective Linear Transformations	Surjective Linear Transformations	Invertible Linear Transformations	Representations	Vector Representations	Matrix Representations	Change of Basis	Orthonormal Diagonalization	Preliminaries	Complex Number Operations	Sets
---------------	---------------	-----------	---------------------------------------	-------	-----------	-------------------------	--------------	-------------------------	--	-------------	------------------------------	--	--------------------------------	------------------------	------------------------	----------------------------------	-----------------------------------	-----------------------------------	-----------------	------------------------	------------------------	-----------------	-----------------------------	---------------	---------------------------	------

We will have theorems in this section that connect nonsingular matrices with systems of equations, creating more opportunities for confusion. Convince yourself now of two observations, (1) we can decide nonsingularity for any square matrix, and (2) the determination of nonsingularity involves the solution set for a certain homogeneous system of equations.

Notice that it makes no sense to call a system of equations nonsingular (the term does not apply to a system of equations), nor does it make any sense to call a 5×7 matrix singular (the matrix is not square).

Example S A singular matrix, Archetype A

Example HISA shows that the coefficient matrix derived from Archetype A, specifically the 3×3 matrix,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

is a singular matrix since there are nontrivial solutions to the homogeneous system $LS(A, 0)$.

(in context)

./knows/example.S.knowl

Example NM A nonsingular matrix, Archetype B