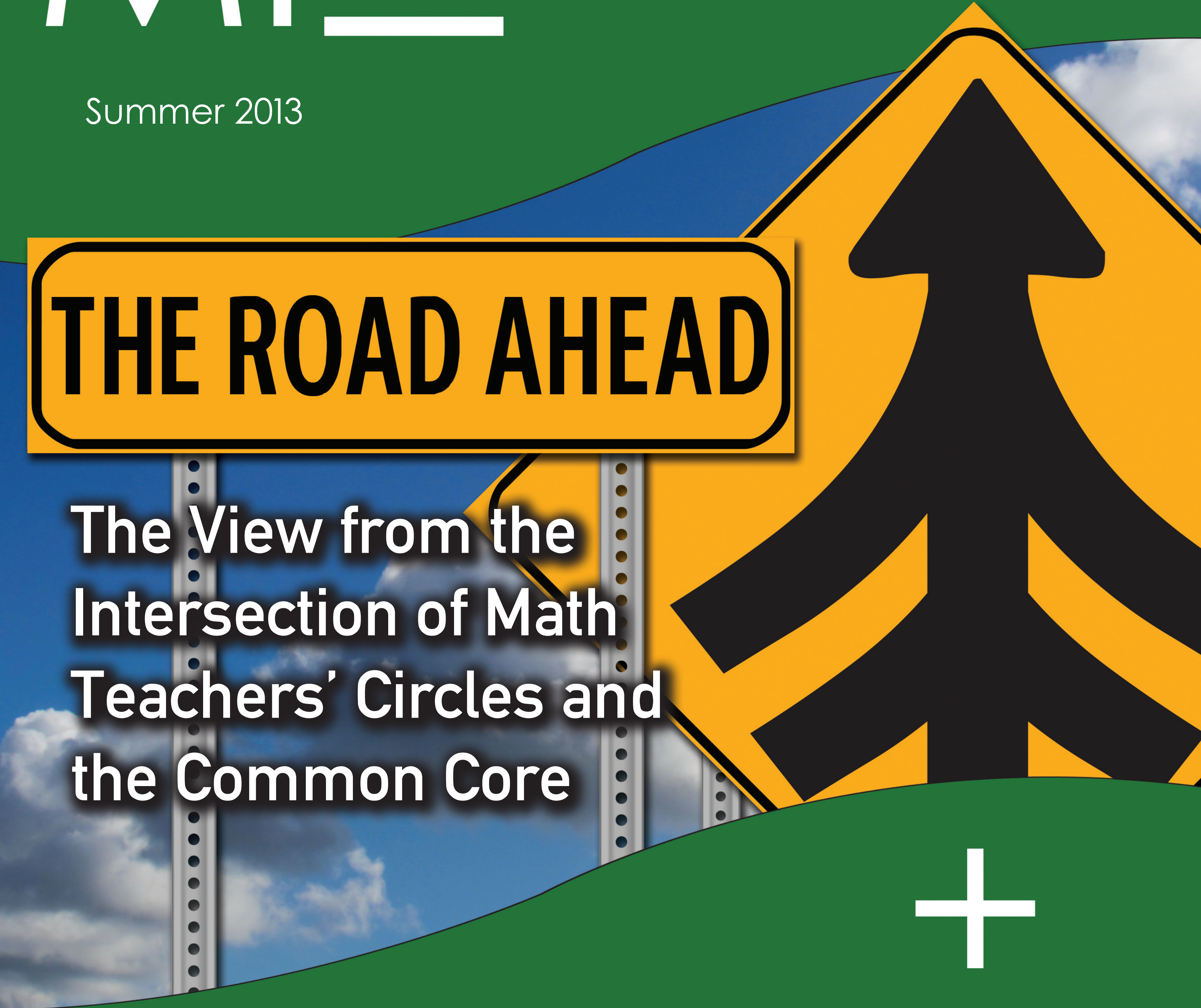
MTCircular

Summer 2013



The View from the Intersection of Math Teachers' Circles and the Common Core

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Escape from Teacher Island Closer Colleagues

Recipe for Success Perfect Session Ingredients

Frogs and Toads Problem Circle

Function Diagrams In Session

A NOTE FROM AIM

Seeing the Familiar in a New Light

Dear Math Teachers' Circle Network,

The Common Core State Standards for Mathematics (CCSSM), with their vision of a deep, coherent mathematics experienced by students as mathematicians, represent a sea change for the teaching of K-12 mathematics in the United States. Since their inception, it has been clear that the CCSSM are deeply compatible with the philosophy of Math Teachers' Circles, and that MTCs could be a meaningful source of support for teachers grappling with the new standards. In this issue's feature article, Michael Nakamaye and Kristin Umland thoughtfully explore intersections between the activities of a Math Teachers' Circle and both the content and practice standards of the CCSSM. I hope that their article helps begin a larger conversation about how MTCs fit in with the implementation of the CCSSM. We would love to hear from MTC leaders and teachers who have explored this question in the context of math sessions or the classroom.

"Escape from Teacher Island," by Jessa Barniol, explores another intersection, between work colleagues and fellow MTC participants. Steve Bartholomew of The Philadelphia School and the Philadelphia Area MTC comments on how MTC meetings help bring out strengths of his colleagues that he might not have known about from staff meetings or the lunch room: "As soon as you start working together on a common problem, you realize how much you can learn from one another."

In a companion piece to his Winter 2013 MTCircular article on good problems, Joshua Zucker lays out some of the key ingredients for creating the kinds of sessions that you'll want to bring your colleagues to.

Seeing the familiar in a new light is also a mathematical theme of this issue. Joshua Zucker looks at function diagrams in this issue's "In Session" and points us toward a number of insights that come from the idea of graphing functions with parallel axes instead of perpendicular ones. As Josh points out, "A new representation will make different aspects more salient; we'll tend to notice new patterns." Thinking about different representations also adds an interesting component to this issue's Problem Corner. We'd love to hear from teachers or MTCs who work on the problem together!

Happy problem solving!

Brianna Donaldson, Director of Special Projects

American Institute of Mathematics

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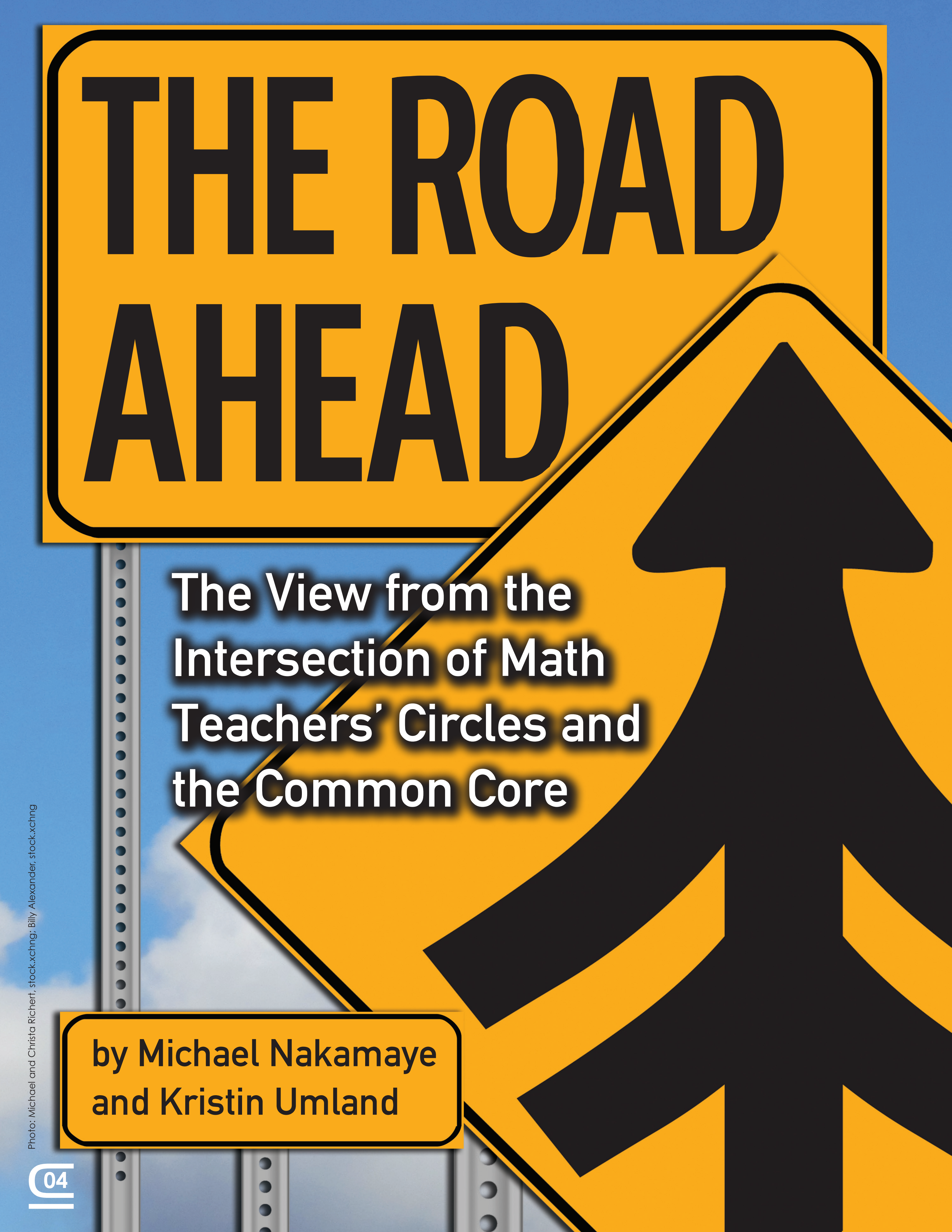

THE ROAD AHEAD

The View from the Intersection of Math Teachers' Circles and the Common Core

by Michael Nakamaye and Kristin Umland

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In 2009, the National Governors Association and the Council of Chief State School Officers brought 48 states together to write "college-and-career-ready" school standards for Mathematics and English Language Arts. Forty-five of those states have adopted the resulting Common Core State Standards for Mathematics (CCSSM) and are currently working to implement them.

Many Math Teachers' Circles have a strong interest in supporting their teachers in the transition to the CCSSM. This interest stems from the compatibility of MTCs with the overall goals of the CCSSM. Specifically, the vision of Math Teachers' Circles is that teachers will experience mathematics as mathematicians, which is a prerequisite for realizing the vision of the Common Core: for students to experience K-12 mathematics as mathematicians.

Beyond a compatible vision, however, what specifically do Math Teachers' Circles already offer that can help teachers understand and implement the CCSSM, and what else can we do to help?

The Structure of the Common Core State Standards for Mathematics

To understand how Math Teachers' Circles might fit in with the Common Core, it's important to know a bit about how the CCSSM are structured.

The three main design principles for the CCSSM are focus, coherence, and rigor. First, the standards focus on fewer mathematical topics and go into more depth, especially in the early grades. In high school, focus is accomplished by organizing the mathematical topics around a small number of unifying principles.

Second, the standards were written to develop a coherent vision of mathematics for students. Rather than treating the subject as a bunch of discrete topics learned in isolation, the standards are designed to reveal mathematics as an unfolding story about interesting and important ideas.

Finally, the standards balance conceptual understanding, procedural fluency, and mathematical applications. This represents a rigorous plan of study in the same way that a well-balanced soccer-training program should have students scrimmage, practice ball skills, and participate in cardiovascular conditioning.

The CCSSM have content standards that are organized by grade level in elementary and middle school and by "conceptual category" (such as "Algebra") in high school. Within each grade level and conceptual category are domains, such as "Operations and Algebraic Thinking" (in grades K-5) or "Seeing Structure in Expressions" (in high school Algebra). Within domains are clusters, and within clusters are the individual standards. Each level of this structure contains information—one might think of the cluster that contains a set of standards as the "convex hull" of the standards within it.

Interwoven with the content standards are eight Standards for Mathematical Practice (see box, above right).

STANDARDS FOR MATHEMATICAL PRACTICE

- **MP1** Make sense of problems and persevere in solving them
- **MP2** Reason abstractly and quantitatively
- **MP3** Construct viable arguments and critique the reasoning of others
- **MP4** Model with mathematics
- **MP5** Use appropriate tools strategically
- **MP6** Attend to precision
- **MP7** Look for and make use of structure
- **MP8** Look for and express regularity in repeated reasoning

MTCs and the Standards for Mathematical Practice

Several of the standards for mathematical practice fit with activities common in MTCs like a hand fits in a glove. For example, MP1 is to "make sense of problems and persevere in solving them." In our MTCs, we give our participants interesting and challenging problems, and they come to expect that, if they bend their mind toward them, they can be successful. MP3 is about developing mathematical arguments and explaining "why," and MP6 is about taking care to communicate precisely about mathematical ideas. MP5 is about "using appropriate tools strategically." Many people interpret this to be only about physical tools (like a compass and straight-edge or a calculator), but it is also about mathematical tools and problem-solving strategies. Finally, MP7 and MP8 capture the complementary concepts of generalizing from specific cases when appropriate and making use of generalizations that have already been established.

To illustrate connections with the mathematical practices more concretely, let's take a couple of examples from actual MTC sessions.

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Take Away

We begin with a game called "Take Away," as formulated by Paul Zeitz:

A set of 16 pennies is placed on a table. Two players take turns removing pennies. At each turn, a player must remove between 1 and 4 pennies (inclusive). The winner of the game is the last one to remove one or more pennies.

The best way to get started with this activity is to play the game. Most people find the initial stages of playing "Take Away" very challenging: after all, with no experience, it is difficult to foresee the consequences of the first few moves.

Once there are only a few pennies left, a clear strategy emerges: for example, if there are between 1 and 4 pennies left and it is your move, then you can win by removing all of the pennies. So this means that you can set as a goal to have between 1 and 4 pennies on the table when it is your turn. On the other hand, if there are 5 pennies on the table and it is your turn, then you have lost because however many pennies you take, your opponent can remove all of the rest.

At this point, it is common to start making a table to work backward and develop a winning strategy. If you are left with 1 through 4 pennies, then you can win, while with 5 pennies, you will lose. If you are left with 6, 7, 8, or 9 pennies, you can win by removing enough to leave your opponent 5 pennies. This pattern continues so that losing situations are those when there are 5, 10, 15, or more generally a multiple of 5 pennies left on the table. When the number of pennies is not a multiple of 5, a winning strategy is to remove enough pennies to leave the opponent with a multiple of 5.

Put another way, the key realization is that the number of pieces you start with can be expressed algebraically as $5n + k$, where k is 1, 2, 3, or 4. Then, the winning strategy is to start by taking away k pennies, and then at each of the following steps, if your opponent takes i, you take $5 - i$, so that your opponent is always left with a losing situation.

This winning strategy is almost always developed by observing what happens toward the end of the game and working backward, thus exemplifying MP1, part of which reads, "consider analogous problems... and try special cases and simpler forms of the original problem in order to gain insight into its solution."

Formulating the winning strategy and showing that it always works requires MP2: the game involves physically manipulating concrete objects (pennies), while the mathematical solution involves remainders of numbers when divided by 5. Hence it is necessary to work back and forth between the context of the game and the decontextualized manipulations of numbers.

Analysis of this game also exemplifies MP8, "Look for and express regularity in repeated reasoning." Indeed, the key to working backward is to identify that, for this game, a group of 5 pennies can be removed without influencing the outcome of the game.

Minimizing a Sum of Distances

The next problem that we will look at comes from Tatiana Shubin:

Suppose A and B are two points on the same side of line l. Find the point P on l so that the sum of the distances $|\overline{AP}| + |\overline{BP}|$ is minimized.

This is a challenging geometric problem, and a full solution requires work relating to many of the standards of mathematical practice. For example, MP3, "Construct viable arguments and critique reasoning of others," specifies that students will "analyze situations by breaking them into cases, and can recognize and use counterexamples."

One way to get started is to consider the special case where the perpendicular line m from A to l contains B (see Figure 1). In this case, P is the point of intersection of m and l. In general, B will not lie on m. Drawing some pictures makes it clear that P must lie between

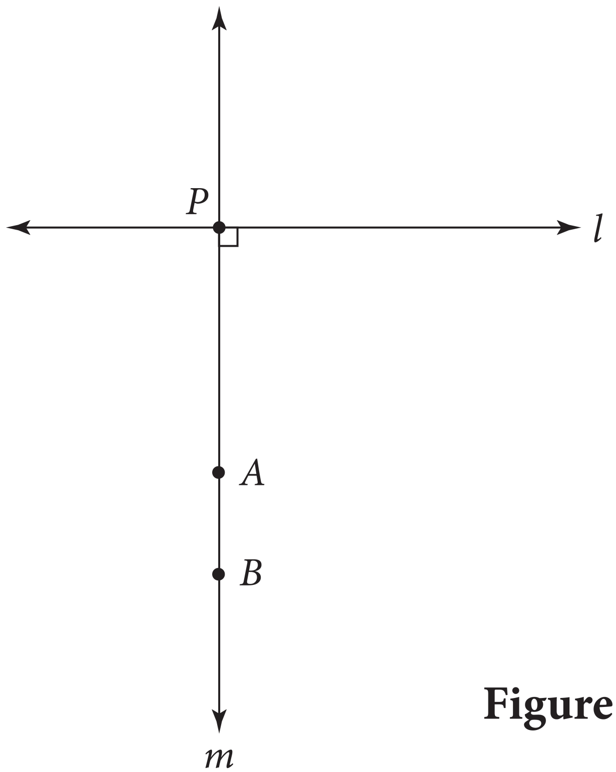

Figure 1

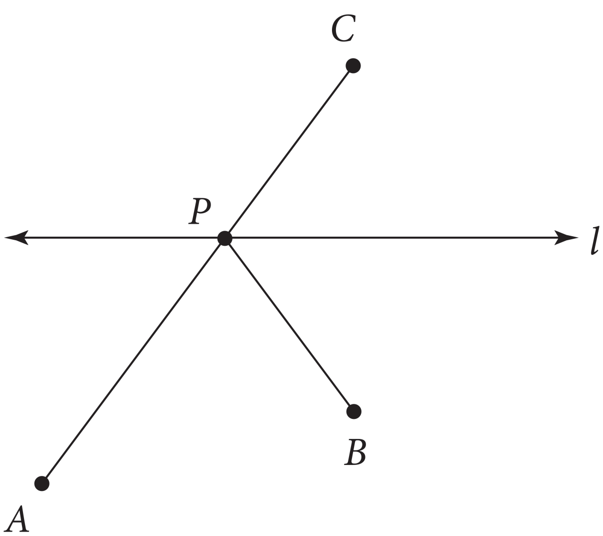

Figure 2

the point where m meets l and the point of intersection of the perpendicular line from B to l.

Progressing beyond this, however, is difficult. One way forward is to examine two points, A and C, that are on different sides of l; in this case, we can use the result which says that the shortest distance between A and C will be achieved by the line segment joining them. Suppose then that we make C the reflection of B about the line l and let P be the point of intersection of $\overline{AC}$ with l (see Figure 2). We know that $|\overline{AC}|$ is the shortest distance between A and C. But $|\overline{AC}| = |\overline{AP}| + |\overline{PC}| = |\overline{AP}| + |\overline{PB}|$, since reflection about l preserves distances between points. Since P minimizes $|\overline{AP}| + |\overline{PC}|$, it must also minimize $|\overline{AP}| + |\overline{PB}|$ as desired. So P is the point of intersection of $\overline{AC}$ with l.

This is a difficult argument that exemplifies MP6, "Attend to precision," as the pictures need to be labeled carefully and referred to accurately. We also see MP7, "Look for and make use of structure," as the key step of working with the reflection of B about l allows us to reduce this problem to a well-known geometric result.

MTCs and the Content Standards

The standards for mathematical practice are the most obvious way for students to experience mathematics as mathematicians, but the structure of the content standards also provides an opportunity for this. The challenge is helping students, teachers, and the people who work with them see and explore the deeper structures in K-12 mathematics. So, it is not just the standards for mathematical practice that need attention, but the natural ways in which the practices can come into play with the specific mathematical ideas laid out in the content standards. In particular, we don't want students or teachers to think that "real" math only starts after high school.

Many successful Math Teachers' Circle activities, like those examined above, contain deep connections not only with the mathematical practices, but with the content standards, too. For example, one content standard exemplified by "Take Away" is the fourth-grade standard 4.OA.5, "Generate number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself." Another is the seventh-grade standard 7.EE.2, "Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related." The geometry problem aligns well with the geometry standard G-CO.B, "Understand congruence in terms of rigid motions," since the key construction is to reflect *B* about *l*. It also aligns with the simpler eighth-grade standard 8.G.A.1, "Verify experimentally the properties of rotations, reflections, and translations," since only the reflection of the point *B* about the line *l* is needed.

In addition to looking for connections between existing sessions and the content standards, we believe that the content of K-12 mathematics could—perhaps should—be a valuable source of inspiration for new sessions. Here are a few examples of ideas in K-12 mathematics that need mathematical explanations:

- Why is addition commutative?
- Why is $a \div b = a/b$?
- Why can you set up a proportion?
- Why is the slope defined by any two points on the same line always the same?
- Why do the properties of exponents work the way they do?

Developing new sessions that are mathematically interesting to all involved but also intentionally touch on some of these ideas would be an excellent way to support the teachers who participate in Math Teachers' Circles, and by extension, their students.

Standing at the Crossroads

The implementation of the Common Core presents a true opportunity for enriching the collaboration among teachers and mathematicians already found in Math Teachers' Circles. We want students and teachers to understand that mathematics is a living, growing subject, and Math Teachers' Circles are a great way for teachers to experience this firsthand. It is equally important for students and teachers to see that this vibrant subject is a natural extension of the mathematics that they do in classrooms. K-12 students should have an opportunity to experience the pleasure of solving a problem for themselves as part of their everyday mathematical work.

In order to make this a reality, we need people who understand the value of solving problems to think about what kinds of problems can help students develop a deep understanding of the core ideas of K-12 mathematics. We believe that the Math Teachers' Circle community is well positioned to contribute to the development of such problems, and that, working together, teachers and mathematicians in Math Teachers' Circles can make a real difference in how the CCSSM are implemented.

Michael Nakamaye is a professor, and Kristin Umland is an associate professor, in the Department of Mathematics and Statistics at the University of New Mexico. Nakamaye is also the leader of the Albuquerque MTC, and both are involved in a National Science Foundation grant to research MTC outcomes.

If you have any ideas or suggestions for MTC sessions or related lessons that connect well to the CCSSM, please send them to circles@aimath.org.

Resources

The Common Core State Standards for Mathematics.

"**Tools for the Common Core Standards**," Bill McCallum's blog.

The Progressions Project for the CCSSM.

Illustrative Mathematics.

Mathematical Games, Paul Zeitz.

The Biggest, the Smallest, Tatiana Shubin.

References

William McCallum, "The Common Core State Standards for Mathematics," presented at ICME 12, Seoul, Korea, July 2012.

Barbara J. Reys (ed.), "The intended mathematics curriculum as represented in state-level curriculum standards: Consensus or confusion?", IAP-Information Age Publishing, 2006.

For links to these resources and more, visit us online at ***http://mathteacherscircle.org/resources/sessionmaterials.html****.*

Recipe for Success

Gathering the Perfect Ingredients for a Great MTC Session

by Joshua Zucker

What makes a good session? As mathematicians, we tend to spend most of our planning time and energy in thinking about what mathematical ideas and strategies we want the participants to work with, and what problems will lead the group to those goals. However, to create and sustain a successful circle, we need to also spend at least as much effort on our thinking about how to use the time we have, how to facilitate interaction and develop community, and what the participants will take away from the experience. Perhaps the most important element is to ensure that every participant feels that their contributions and efforts are worthwhile and valuable.

Leadership

An important part of the goal of MTCs is to bring together mathematicians and middle school teachers. However, there's often still some distance and separation between these two groups. One way to break this down is to have sessions that are co-led by a mathematician and a middle school teacher. Often, the middle school teacher leads the introduction of the problem and facilitates the early discoveries about it, and then the mathematician helps guide the group to see the connections to deeper strategies or to other mathematical topics. This collaboration builds a stronger mathematical community. The middle school teachers are more in control of the process of mathematical discovery, and the mathematicians learn pedagogical strategies and gain insights into how others might experience a problem encountered for the first time.

Efficiency

Starting at the advertised starting time is important, to make everyone feel their time is valued. If you're going to open with some casual socialization before starting on the math, give people that schedule ahead of time so they know what they're coming for.

The Casco Bay MTC suggests one way to make the start feel important while still allowing latecomers to be brought into the conversation: they warm up with an easier problem, or a game that's on a similar theme, such as a parity game for a session that explores other applications of parity. That way people arriving early can get engaged in the mathematics and later can enjoy discovering the connection between the opener and the main session. Meanwhile, people who arrive late don't miss out on the meat of the session, just the appetizer. Another method is to start with the literal meat instead of the metaphorical, by opening with dinner and some social time, so late arrivers may miss out on the food but not the mathematics!

Attendance

Small groups can work well, but there does seem to be some minimum size, around 10, to make the session feel like a success and make the work you put into planning feel worthwhile. Recruitment can be the hardest part of organizing a session, but it can also be the most rewarding as you bring new people into the culture of problem solving. Summer immersion workshops may be one of the best recruiting tools, but you can make progress with a special one-day intensive workshop sometimes, too. Another great way to help the group grow is to get teachers to bring their friends and colleagues. This not only brings more teachers into your group, but also gives more opportunities for teachers to continue thinking about the mathematics and to collaborate on ways to bring MTC problems into the classroom.

A more formal RSVP process may also help increase attendance. People who reply and register on a simple Google Docs form may be more likely to honor that

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commitment, and it's also easier for organizers to send them repeated reminders if they've registered. Some groups, such as Cincinnati, use a Facebook page for their group and announce their events that way, which gives a very easy way of collecting replies.

Participation

Perhaps the most vital element of a good MTC session is that everyone attending should feel like an important participant. The Twin Cities MTC reminds us that a good topic should "have different entry points for those with different backgrounds" and of course all of us understand that a good MTC problem should have a low threshold and high ceiling so that everyone can start and nobody gets bored.

Even though problem selection was discussed in the Winter 2013 issue of MTCircular, it's worth reminding ourselves that less is more: give the participants work to do, things to investigate. Resist the temptation to do more problems and tell them more about the math and the connections. Wait until they need it, wait until they want the mathematics, because they know what they need to continue making progress.

Make sure you have an early punch line so everyone sees at least one important bit (a strategy, a new mathematical idea). The rest can be in a handout that you give at the end of the session, so that you know they can still explore it and you feel less pressured to include those ideas during the session. The handouts can also show extensions to other mathematics or related results, or give a classroom-usable idea on a related topic. Post these on the web so that people at other MTCs can make use of them! The national organization is working on ways to make these handouts easier to find and use, so if you have ideas on how we can do that, please share them with us.

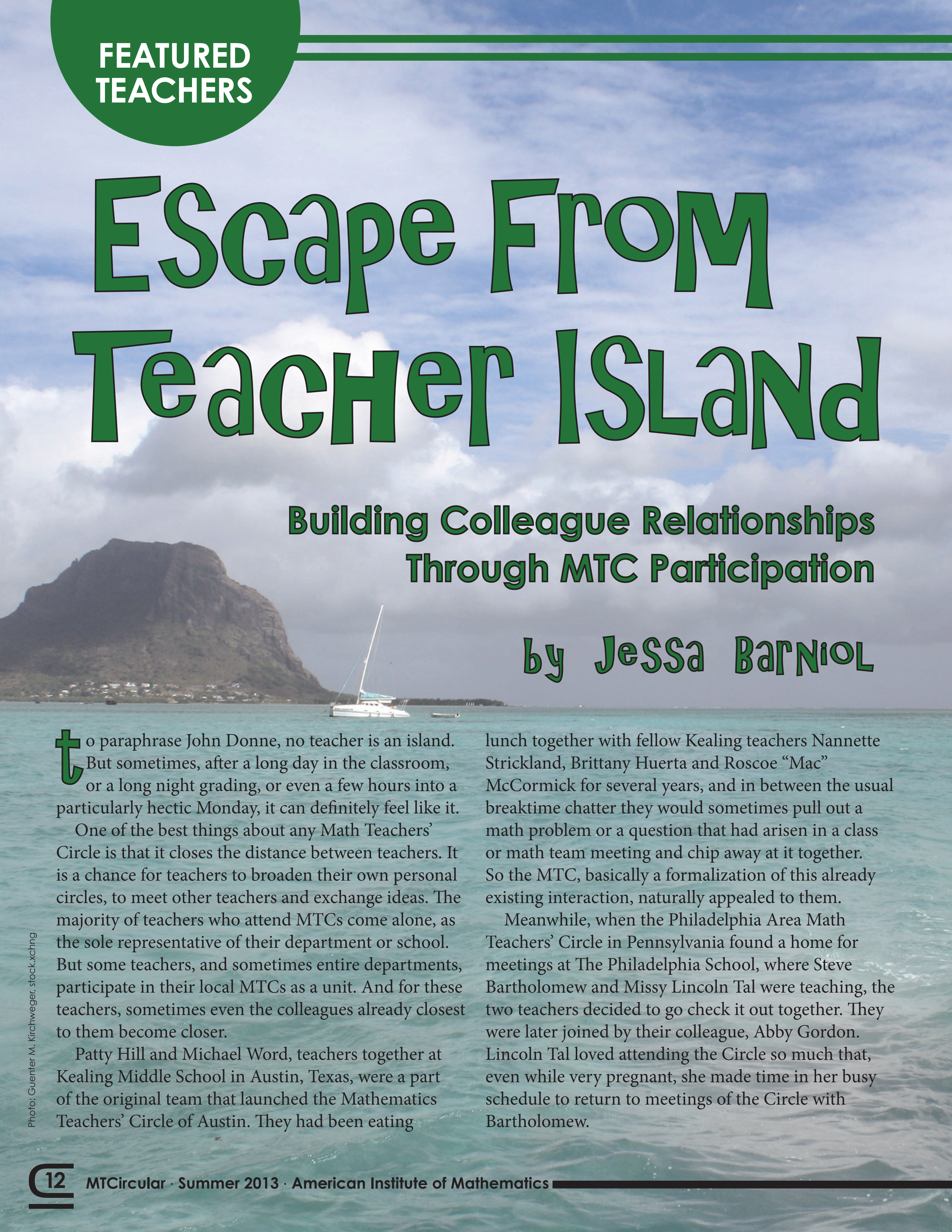
Grouping

Some circles make a deliberate effort to bring new people into the community and to develop new relationships among the attendees. During the course of the session, they form teachers into groups in different ways, sometimes specifically requiring that teachers group with people they haven't met or haven't gotten to know in previous sessions. This is especially important to make the group more welcoming to those who haven't been to a summer workshop, where some participants have already developed strong connections. It can be similarly important for groups like Central Nebraska's, where there is a mix of pre-service and in-service teachers.

Pedagogy

MTC sessions model the pedagogy of problem solving so that teachers can experience it as learners. Many MTCs spend some time discussing pedagogy more explicitly as well. Hawaii, for example, observes two distinct phases in their meetings. First, the participants function as solely mathematical learners, and find that they can stay in that mode more easily when they know that there will be time later for them to reflect on the experience as teachers. The discussion in this second phase includes connections to the Common Core and how to incorporate these problems or related ones into their classrooms. among other things.

Above all, remember that a good session should model the style and practices that we want teachers to use with their middle school students. We want to develop the teachers' ability to use mathematical tools and problem-solving strategies so that they can lead their students in the same direction. Providing clear labels for a small number of key elements of each session can go a long way toward enabling teachers to, in turn, bring the same elements to their students.



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FEATURED TEACHERS

Escape From Teacher Island

Building Colleague Relationships Through MTC Participation

by Jessa Barniol

To paraphrase John Donne, no teacher is an island. But sometimes, after a long day in the classroom, or a long night grading, or even a few hours into a particularly hectic Monday, it can definitely feel like it.

One of the best things about any Math Teachers' Circle is that it closes the distance between teachers. It is a chance for teachers to broaden their own personal circles, to meet other teachers and exchange ideas. The majority of teachers who attend MTCs come alone, as the sole representative of their department or school. But some teachers, and sometimes entire departments, participate in their local MTCs as a unit. And for these teachers, sometimes even the colleagues already closest to them become closer.

Patty Hill and Michael Word, teachers together at Kealing Middle School in Austin, Texas, were a part of the original team that launched the Mathematics Teachers' Circle of Austin. They had been eating lunch together with fellow Kealing teachers Nannette Strickland, Brittany Huerta and Roscoe "Mac" McCormick for several years, and in between the usual breaktime chatter they would sometimes pull out a math problem or a question that had arisen in a class or math team meeting and chip away at it together. So the MTC, basically a formalization of this already existing interaction, naturally appealed to them.

Meanwhile, when the Philadelphia Area Math Teachers' Circle in Pennsylvania found a home for meetings at The Philadelphia School, where Steve Bartholomew and Missy Lincoln Tal were teaching, the two teachers decided to go check it out together. They were later joined by their colleague, Abby Gordon. Lincoln Tal loved attending the Circle so much that, even while very pregnant, she made time in her busy schedule to return to meetings of the Circle with Bartholomew.

Then, at Cope Middle School in Bossier City, Louisiana, teachers Lisa Pumphrey and Jacob Hesselschwardt were invited to a meeting of the North Louisiana Math Teachers' Circle by their middle school's math supervisor, a founder of the Circle. They continue to participate in MTC meetings together to this day, and often brainstorm ways to adapt MTC problems to their respective classrooms.

"I think that having familiar faces there, other teachers from your own school, enhances the MTC experience," Hesselschwardt said. "It encourages you to be an active participant. You are already a little more comfortable, and you are already aware of each others' positive strengths. So you can encourage and help one another. It makes it impossible to be a wallflower."

Attending a Circle together might also help teachers to see their own colleagues in a different light.

"You might have one person who's really intuitive, someone else who is incredibly organized, and perhaps a brainstormer with revolutionary, out-of-the-box ideas," Steve Bartholomew said. "You might never get to see that side of them in staff meetings or the lunch room. But as soon as you start working together on a common problem, you realize how much you can learn from one another."

From her prior friendship with her own fellow teachers, Patty Hill demonstrates how this enhanced knowledge of your colleagues can be helpful as a teaching resource.

"Michael [Word] is the geometry whiz in our department," Hill said. "My students know that whenever they stump me in a geometry question, I'm going right across the hall to ask him for input."

Lisa Pumphrey acknowledges that many middle school teachers can feel like fish out of water, particularly in their first few years teaching.

"I may have an elementary teaching background, and someone else may have a high school teaching background," Pumphrey said. "It is entirely possible, and even probable, to be an eighth grade math teacher who was not a math major. Learning to collaborate with other teachers, particularly with your own coworkers, helps you through that. The MTC

Reel 'em In

Here, the teachers offer their best advice for getting groups of colleagues interested in attending an MTC together in the first place.

"Hold an MTC session at your school. Either your school can host a general regular meeting of your MTC, or you can have one of your MTC contacts come to your school to lead a session as a part of an in-service professional development day. It's hard for teachers to say no when the MTC comes to you the first time, especially if food is involved. We held a few mini-MTC sessions at our school to test the waters before launching the MTC, and I think that is a large part of why so many of our teachers are so invested in the Circle." **– Patty Hill**

"Just get people to go once. When they see how great the meetings are, and how much value they add to the classroom, they will be hooked."
– Jacob Hesselschwardt

"It takes a certain type of teacher to be interested in Math Teachers' Circles. Find like-minded teachers and get them on board. Find the people who would actually be most interested and invite those people." **– Michael Word**

"Have your school's math chair or someone else in authority encourage your department to go together. Even a gentle nudge from a higher-up is sometimes all the encouragement some people need." **– Steve Bartholomew**

"Just start small. Bring one colleague along, then another. Build one-on-one relationships with your colleagues, both at school and in the Circle. Then introduce your new friends to each other. People will be more likely to attend and keep attending if they are excited about seeing their friends."
– Nannette Strickland

can help connect you with someone to show you the ropes. Even better, it can help you forge a connection with someone from your own workplace."

However, there can also be downsides to attending the MTC together with teachers from your own school. Nannette Strickland said that often coworkers have such a similar frame of reference that, even as a collective group, they have much to learn from an outsider. She maintained that it is important to be mindful to use your fellow teachers as constructive team members and not as a safety net or a wall to keep you inside your comfort zone.

"At one of our earliest MTC meetings, which was all teachers from our own school, we were working together on a problem with a hypothetical line of lockers," Nannette Strickland said. "We were to open every second locker, then open or close every third locker, and so on, and figure how to find how many lockers were open at any given time. We were having a hard time finding a way to represent the problem mathematically, when a 6th grade special education teacher from our school, a person outside our usual frame of reference, recommended we use a set of manipulatives and simply flip them over each time the locker was 'opened' or 'closed,' so we could start by getting a good visual representation of the problem. Her students responded best to that type of learning, but we were all, collectively, so far removed from that style that we had failed to even consider it. Sometimes it takes working with different people to remember that there are other ways of doing things."

Even though many of the teachers were already good friends and close colleagues before attending their local MTCs together, they mentioned that the change of scenery improved their interactions and collaborations.

"Our math department was already meeting once a week, but the MTC just gave us something new to talk about," Steve Bartholomew said. "It injected new life into our interactions."

Jacob Hesselschwardt insists that this benefit will occur whether or not other teachers from the same school are able to attend together.

"It is really helpful to have this one evening a month where you change your scenery and perspective, whether it's just you as an individual teacher, or a number of teachers from your department," Hesselschwardt said. "If you put yourself in a new situation, you will learn something. If you put your existing group within a larger group, you will learn something. You will learn what works and what doesn't."

Either way, participating in a Circle can help teachers broaden their perspective, become better collaborators, and, yes, even get off the proverbial island.

"We are all math nerds at heart," Patty Hill said. "When you've been teaching the same class for 15 years, and you're facing the teach-to-the-test focus that you find at basically every school nowadays, you lose touch with your inner mathematician. You might need to be reminded why you love it. Working with great colleagues and participating in the MTC both accomplish this. But if you have both, it's even better."

Left, Nannette Strickland (fourth from right) and Roscoe "Mac" McCormick (second from right) of Kealing Middle School and the MTC of Austin explore the geometry of the card game Set with a physical model. Right, Strickland (far left) and McCormick (upper right) use a more traditional pencil-and-paper approach to new MTC topics.

Top Photo: Guenter M. Kirchweger, stock.xchng; MTC Photos Courtesy of MTC of Austin

desJardins/Blachman Fund Donates to MTCs

The desJardins/Blachman Fund recently donated $24,000 for continuing grants to MTCs that have been meeting for at least one year. These grants will support MTCs that have successfully launched and help them continue their activities. In many cases, MTCs have been able to find funding initially, but they have encountered difficulties obtaining ongoing funding.

"Thanks to this generous donation from the desJardins/Blachman Fund, a number of established MTCs will be able to continue their important work," said Brian Conrey, Executive Director of AIM.

desJardins/Blachman Fund grants were awarded to:

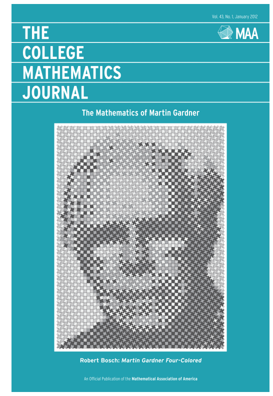
- Casco Bay Math Teachers' Circle (Portland, Maine)
- Fairfield County Math Teachers' Circle (Fairfield, Connecticut)
- Mathematics Teachers' Circle of Austin (Austin, Texas)
- Metro Atlanta Math Teachers' Circle (Kennesaw, Georgia)
- New York Math Circle (New York City, New York)
- North Louisiana Math Teachers' Circle (Shreveport, Louisiana)
- Omaha Area Math Teachers' Circle (Omaha, Nebraska)
- Philadelphia Area Math Teachers' Circle (Philadelphia, Pennsylvania)
- San Benito County Math Talks (Hollister, California)
- San Diego Math Teachers' Circle (San Diego, California)
- Sioux Falls Area Math Teachers' Circle (Sioux Falls, South Dakota)
- Twin Cities Math Teachers' Circle (St. Paul, Minnesota)
- Winston-Salem Math Teachers' Circle (Winston-Salem, North Carolina)

Math for America Awards MTC Seed Grants

Congratulations to the following MTCs who were awarded Math for America seed grants through AIM:

- Capital City Circle (Jackson, Mississippi)
- Cincy Math Circle (Cincinnati, Ohio)
- East Texas Math Teachers' Circle (Nacogdoches, Texas)
- Heartland Math Teachers' Circle (Olathe, Kansas)
- Math Teachers' Circle @Stony Brook (Stony Brook, New York)
- Rochester Area Math Teachers' Circle (Rochester, Minnesota)
- San Joaquin Math Teachers' Circle (Stockton, California)
- South Central Texas Math Teachers' Circle (Seguin, Texas)
- South East Ohio Math Teachers' Circle (Athens, Ohio)
- Thousand Oaks Math Teachers' Circle (Thousand Oaks, California)
- Mid-Hudson Math Teachers' Circle (Annandale-on-Hudson, New York)

MTC Session Leads to Journal Article



After investigating some open questions about the card game Set in a Spring 2011 session, AIM MTC participants in Palo Alto, California, wrote up their results and submitted them to the **College Mathematics Journal**. The 13-author article, entitled "Sets, Planets, and Comets," is scheduled to appear in the September 2013 issue of CMJ. The article explores some geometric properties of the Set card deck and introduces a new variation on the game invented by the MTC participants. Look for more details in the next MTCircular!

Member Circles Plan Summer Workshops

The South East Ohio MTC leadership team.

Over a dozen member Circles are planning summer workshops for their participants. Please check out the announcements page at **http://www.mathteacherscircle.org/workshops.html** for details.

Funding for these summer workshops has been provided by a variety of sources and has been secured through the efforts of each local group. The workshops of the South East Ohio MTC, Metro Atlanta MTC, and Omaha MTC will be primarily supported by state Title IIA Improving Teacher Quality grants, funded through the U.S. Department of Education. U.S. Department of Education funds will also support the East Texas and Heartland MTCs.

Other MTCs are receiving funds from businesses and foundations in their communities, including State Farm Insurance (Northern Colorado MTC), the Mikkelson Foundation (Rocky Mountain MTC), and the Winston-Salem Community Foundation (Winston-Salem MTC). Workshops will also be supported in part through grants awarded by the Mathematical Sciences Research Institute and National Association of Math Circles, as well as by Math for America seed grants and desJardins/Blachman Fund continuing grants awarded through AIM (see previous page for lists of recipient MTCs). Congratulations to all these Circles and best of luck with all your future plans!

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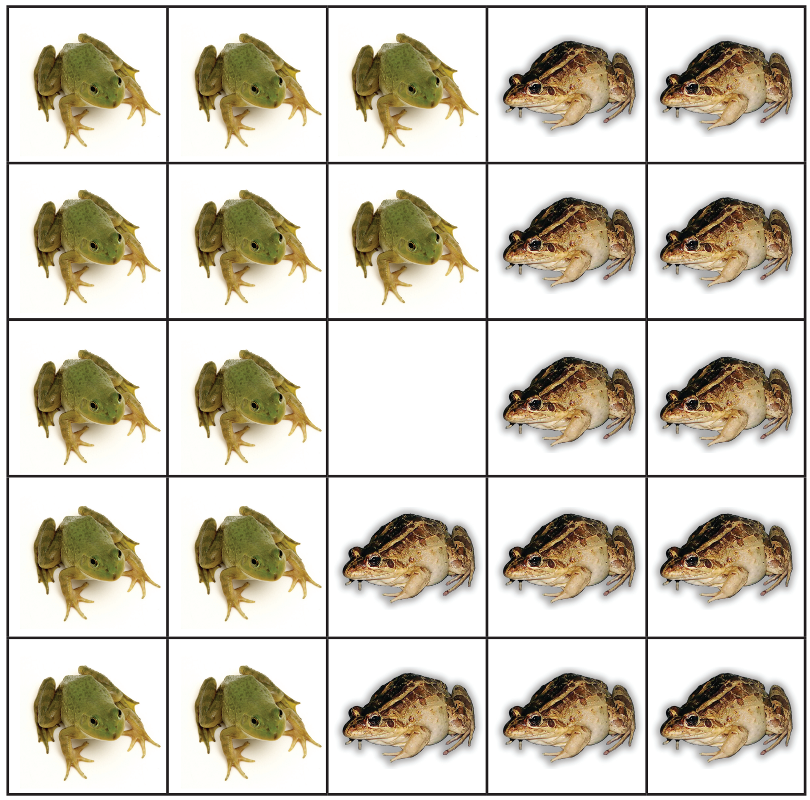
BY JOSHUA ZUCKER

Problem Circle Frogs and Toads

In this game, frogs can move right or down. Toads can move left or up. A frog or toad can slide into an adjacent empty space. They can also jump over exactly one of the other kind of animal to land in an empty space. To win, exchange all the frogs with the toads. How many moves does it take? How can you represent your solution? What if the board were 7 by 7, or $(2n + 1)$ by $(2n + 1)$?



Photos: Jose Carli, stock.xchng; Adrian Gutierrez, stock.xchng

Links and Resources

Cut the Knot, "**Toads & Frogs Puzzle**" and "**Toads & Frogs Puzzle 2D**."

NRICH, "Frogs."

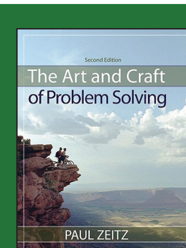
Puzzles.com, "Jumping Coins."

Anthony Fernandes, et al., "**Mathematics Teachers Circle Around Problem Solving**," Mathematics Teaching in the Middle School, Sept. 2011. Article available upon request to **Anthony.Fernandes@uncc.edu**.

For links to these resources and more, visit us online at ***http://mathteacherscircle.org/resources/sessionmaterials.html***.

WIN A FREE BOOK!

Got an answer to this issue's Problem Circle? Send it in and you could win! MTCircular is offering a copy of any of the books listed at http://mathteacherscircle.org/resources/general.html or http://mathteacherscircle.org/resources/sessionmaterials.html to the best solution or partial solution. Send your answers to problemcircle@aimath.org for consideration.



Simple, or Impossible?

$$x^x \cdot y^y = z^z$$

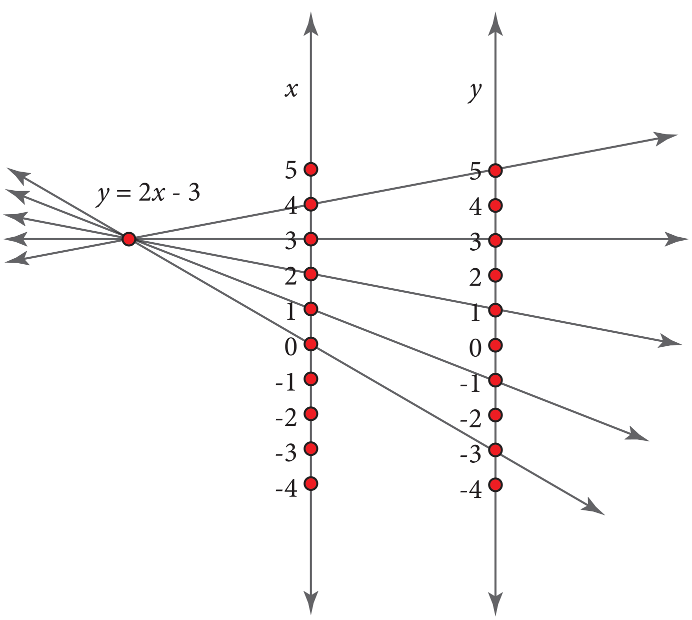
Last issue, we asked for integer solutions to the equation $x^x \cdot y^y = z^z$ for which all the variables are greater than 1. One such solution is $x = 2^{12} \cdot 3^6$, $y = 2^8 \cdot 3^8$, and $z = 2^{11} \cdot 3^7$. Our offer of a free book still stands for the best additional solution sent to **problemcircle@aimath.org**.

In Session: Function Diagrams

A Session by Henri Picciotto

Some of the best sessions come from what-if questions. What if Euclid's fifth postulate weren't true? What if only even numbers had been invented?[1] What if we graphed with parallel axes instead of perpendicular ones? What would that even mean?

Let's begin by graphing $y = 2x - 3$. We do this in the ordinary way: find some pairs of values that work, like (0, −3) and (1, −1) and (2, 1) and (3, 3) and (4, 5). But now, instead of plotting a point for each of these pairs, we locate the x value on the x axis and the y value on the y axis, and then connect them with a line. If we tried to plot all the pairs, we'd end up with infinitely many lines and it would look like a solid blob of ink, so we plot just a few pairs to illustrate the pattern.



And look at what we discover! The graph of the equation $y = 2x - 3$ is a point: the point where all the lines (0, −3), and (1, −1), and so on, meet. You might enjoy finding a proof that all these lines really do meet at a point, called the focus. More importantly, before you read farther, graph a few other equations on parallel axes. It might be convenient to place your axes 6 units apart. Henri Piccotto's first handout[2] might also make some good practice here, especially with the teachers' notes that he gives[3].

There is a big principle at work here: duality. In many geometries, there are interesting transformations that exchange points and lines, and replace points being on the same line with lines passing through the same point as we see in this example. Using that idea, can you figure out how to use function diagrams to solve a system of equations?

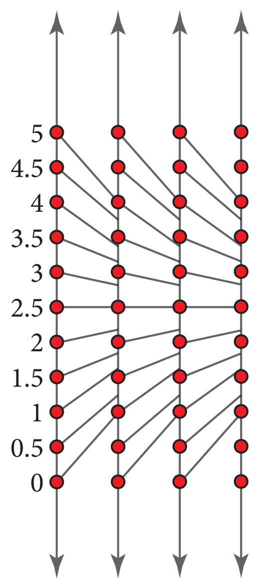
Perhaps more importantly, we now have a new representation for our old friend $y = mx + b$. A new representation will make different aspects more salient; we'll tend to notice new patterns. For me, the big difference with these function diagrams is that I now notice the horizontal lines in them: that is, where the input x value is equal to the output y value. I would be prone to write our sample equation as $y - 3 = 2(x - 3)$ instead, to emphasize that fact. To practice writing equations for different function diagrams, you can download another handout[4], and also enjoy the teachers' notes that go with it[5].

The similar triangles that you can see in this representation also give a meaning to the m in our equation: It's the magnification! Each unit on the x-axis gets magnified by a factor of m as we go to the y-axis. It may take a while to get a gut-level appreciation that the magnification stays constant even while the "slope" of these lines is varying.

A new representation can also make certain aspects of our equation $y = mx + b$ easier to see and understand. For example, consider the amount of medication

BY JOSHUA ZUCKER

flowing in the bloodstream of someone who takes one antibiotic pill per day. Perhaps the body removes 40 percent of the medicine in a day, and then 1 more unit is added, so with y being the next day's amount and x being the current amount, we have $y = 0.6x + 1$. If we want to know how much of the medication will be in the bloodstream a few days later, we can use an iterated function diagram like this one:



The function is the same every time, but since we're only plotting a few of the (input, output) pairs, you'll have to imagine the way things flow in the spaces between the lines as we apply the function repeatedly.

Now we can see that, no matter where you start, after a few days you'll already be pretty close to 2.5 units of medication. And now you know why some antibiotics have you take two pills the first day, too!

This representation also lends itself to the composition of functions. A few such diagrams and some thought about magnifications will lead to an easy, visual proof of the chain rule for derivatives.

You may also make some interesting discoveries of your own by graphing quadratics. There's an amazing thing to notice in the graph of $y = 1/x$, too. Enjoy!

References

1. "**Primes in Evenland**," New York Times Wordplay blog.
2. "**Nine Function Diagrams**," Math Education Page.
3. "**Function Diagrams**," by Henri Picciotto, Math Education Page.

   (Author's note: See also the additional links at the bottom of the web page. The brief history and bibliography there cite many sources for these ideas. If I recall correctly, I first learned about function diagrams at a conference session led by a couple of teachers from the Illinois Math and Science Academy.)
4. "**Sixteen Function Diagrams**," from *Algebra: Themes, Tools, Concepts* by Anita Wah and Henri Picciotto, Math Education Page.
5. "**Focus on Function Diagrams: Teacher Notes**," from *Algebra: Themes, Tools, Concepts* by Anita Wah and Henri Picciotto, Math Education Page.

For links to these resources and more, visit us online at ***http://mathteacherscircle.org/resources/sessionmaterials.html***

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For a list of Session Materials featured in this issue, visit http://www.mathteacherscircle.org.

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