MTCircular

Dear MTC Network,

Welcome to the first issue of *MTCircular*! This newsletter will provide a forum for sharing ideas and celebrating the accomplishments of our growing community of MTCs.

MTCircular will appear several times per year. Each issue will include a featured MTC, an interview with a teacher, a column highlighting a session from our website, news and upcoming events, and more.

We welcome your feedback and suggestions at circles@ aimath.org. I hope you enjoy the newsletter!

Sincerely,

Brianna Donaldson
Director of Special Projects
American Institute of Mathematics

Newsletter Contents

American Institute
of Mathematics

Featured MTC: The Lincoln Area Math Teachers' Circle

Location: Lincoln, NE
Founded: 2007
Sponsors: University of Nebraska-Lincoln (UNL) Center for Science, Math, and Computer Education; Lincoln Public Schools (LPS)

The founders of the Lincoln Area MTC attended the very first "How to Run a Math Teachers' Circle" workshop, held at AIM in 2007. Four years later, the LAMTC is still going strong, with academic-year meetings attended by an average of 20-25 participants. We caught up with leaders Steve Dunbar (UNL), Julie Kreizel (LPS), and Anne Schmidt (LPS) for an inside view of their Circle.

Why did you start the LAMTC?

Julie Kreizel: Through Math in the Middle [an Institute Partnership funded by the National Science Foundation to develop middle school teacher leaders in mathematics], we had developed quality relationships with the other students and the instructors. We wanted these relationships to continue. MTC seemed to match exactly what we were looking for.

Anne Schmidt: From Math in the Middle, we got to appreciate the ability to work together and have conversations about the problems. It was like being in a foxhole in a war. You know you have each other's back and will help each other through. We wanted to continue the experience without the evaluative part.

Steve Dunbar: The instructors for Math in the Middle learned a lot about mathematical knowledge for teaching and found it interesting. We also found the relationships with the teachers valuable and wanted this to continue.

The LAMTC has been extremely successful in involving teachers in co-leading sessions. Do you have any tips for other MTCs who want to involve teachers in this way?

SD: A good source of topics has been research papers written by Math in the Middle teachers, as they are re-

LAMTC leaders at AIM. From left: Cheryl Miner, Steve Dunbar, Julie Kreizel, Sue Graupner, and Anne Schmidt.

quired to do a paper on a mathematical topic of considerable depth. We've found that this process helps teachers develop a real confidence to tackle a problem outside of their curriculum. We tell them, "You don't need to know everything about this. We'll find someone who can provide backup." And it really goes both ways. Teachers find neat ways to add to the presentation where my tendency might be to just talk through something.

Anne, you recently co-led a session on the Water Jug Problem. Could you describe your approach?

AS: Delise Andrews and I worked with Ira Papick, a mathematician at UNL, on this session. The basic problem is, you have a 3-gallon jug and a 5-gallon jug, and you want to get exactly 4 gallons of water. We ended up making wooden blocks for everyone that were various lengths, and using a ruler to measure off what lengths we could make given certain starting lengths. The group examined several cases and worked out how in general you can figure out this type of problem using the Euclidean algorithm.

SD: The Jug Problem clips from *Die Hard* were great!
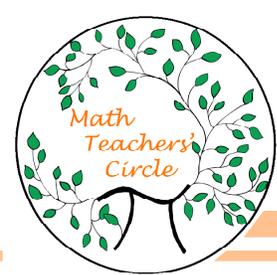
What do you think makes for an especially effective MTC session?

SD: We've found that having things to make, do, create—"freebie manipulatives"—really helps. For example, Stephen Hartke did a session on the Instant Insanity puzzle, and he had everyone create Instant Insanity cubes from a construction paper diagram**.**

JK: Really, the best sessions follow the idea of a quality lesson plan and model how to interact with students. With manipulatives, we want to go beyond playing with them to actually using them to gain a deeper, better understanding of math. This mirrors what we want teachers at all levels to take back to the classroom.

Several UNL graduate alumni have gone on to start their own MTCs after joining the faculty at another institution. Could you talk a little about how graduate students get involved?

SD: Students see there is interesting math and there are interesting people to work with, pre-service and in-service middle school teachers. They get hooked. I often look first to the graduate students to partner with teachers on a MTC session.

I should also mention that the involvement of these students in the LAMTC is a plus in the job interview process. Several of these grads have been hired in part because of their experience working with teachers and their ability to get their own MTC going.

What advice do you have for MTC leaders about keeping the momentum going for multiple years?

AS: It's essential to have good session leaders and topics, a good venue, a positive relationship between spending time at the meetings and the classroom, and word of mouth to advertise the meetings.

JK: Establishing a professional and social network is key so that you have a good core that reaches out. Mealtime together at the start is important, and Anne always brings chocolate for the end of the meetings.

The LAMTC works out the math behind Conway's Rational Tangles.

SD: The LAMTC is a real partnership between the university and the schools. Everyone is part of the whole program. Some people go to book club. I view MTC as my book club. It provides social interaction, then a program that is fun and interesting and different from most things that I do every day.

Links and Resources

The Lincoln Area MTC
http://www.scimath.unl.edu/mathteacherscircle/

Jug Problem video clips from *Die Hard*
http://www.scimath.unl.edu/mathteacherscircle/#tab_lincoln_2009

"The Jug Band" (Elgin Johnston)
http://www.mathteacherscircle.org/resources/sessionmaterials.html

"Three Jugs" (Joshua Zucker)
http://www.msri.org/web/msri/static-pages/-/node/211

Instant Insanity Cubes
http://www.scimath.unl.edu/mathteacherscircle/files/lincoln/2009/InstantInsanity_cubes.pdf

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Problem Circle

by Joshua Zucker

A recent talk of Tatiana Shubin's kept me on my toes by presenting several problems of the form "Is it possible to ...?" Several times, I tried to prove something impossible when I should have been trying to construct it, or vice versa. For example:

- Is it possible to paint each point in the plane, using three different colors, so that every straight line in the plane has exactly two different colors? (The black lines at right illustrate places where the shown painting scheme fails.)

- Can you paint a circle (not a disk) with two colors so that no isosceles triangle has three vertices of the same color? What if you can use three colors? A thousand colors?
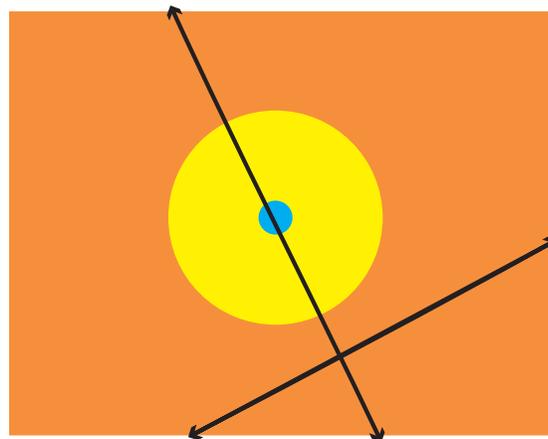
Links and Resources

"Have Colors - Will Paint" (Tatiana Shubin)
http://www.mathteacherscircle.org/resources/sessionmaterials.html

You can send your (or your MTC's) solutions to Joshua Zucker at problemcircle@aimath.org*. Selected solutions will be posted with the next issue of* MTCircular*.*

MTCs Make a Difference

Research on MTCs has recently begun to provide evidence about how the program affects middle school teachers. Some highlights of this ongoing research include:

Diana White (University of Colorado Denver) found that middle school teachers who participated in a weeklong intensive MTC workshop in Summer 2010 significantly increased their scores on a standard test measuring mathematical knowledge for teaching. Previous research by Heather Hill (Harvard University) and her colleagues has linked higher mathematical knowledge for teaching scores with increased student achievement.

In a nationwide survey conducted in Fall 2010 by Diana White and Brianna Donaldson (American Institute of Mathematics), MTC participants reported increased enthusiasm for mathematics and use of interactive, student-centered problem solving in their classrooms. Many said the program has enabled them to see themselves as mathematicians. Teachers also reported that participating in a MTC has increased their belief that all of their students are capable of doing mathematics.

MTCircular will report more details on this and other research in our fall issues. Write-ups and presentations documenting research on MTCs are available on the MTC Network website at http://www.mathteacherscircle.org/about/background/outcomes.html

MTC of Austin Wins Outreach Award

The Math Teachers' Circle of Austin (MTCA) recently won the 2011 College of Natural Sciences Outreach Innovation Award from the University of Texas at Austin. This award, established by the Dean of the College in 1999, recognizes "an individual or group within the College of Natural Sciences who has represented the College in outreach to the community and whose outreach efforts go above and beyond the day-to-day responsibilities of their job."

MTC of Austin leaders at AIM. From left: Adriana Sofer, Patty Hill, Altha Rodin, and Jason Ermer. Not pictured: Michael Word.

The award was presented in April to Drs. Altha Rodin and Adriana Sofer, who are Senior Lecturer and Lecturer, respectively, in the Department of Mathematics at UT Austin. "We see this award as an endorsement of the Math Teachers' Circle of Austin," said Sofer, noting that an additional benefit of the award has been to raise awareness of the program among others at the university. Department of Mathematics Chair William Beckner, who nominated Rodin and Sofer for the award, said, "Altha and Adriana had a dream—to affect teaching in the classroom—and this recognition highlights their success."

Rodin and Sofer, together with the rest of their team, Jason Ermer (UT Austin), Patty Hill (Kealing Middle School), and Michael Word (Kealing Middle School), began planning MTCA in 2009, when they attended a "How to Run a Math Teachers' Circle" workshop at AIM. After successfully obtaining support from the Educational Advancement Foundation and the UT Austin Department of Mathematics, the group held a week-long summer immersion workshop in August 2010 followed by seven evening meetings during the 2010-2011 academic year. This year, MTCA has explored topics such as patterns of carrying in addition, Moser's circle problem, the geometry of the card game SET, inductive thinking, and asking "What-if-not" to generate new problems.

Evaluations of the program have been positive, with teachers noting the collegiality of the meetings and the strong impact on their approach to teaching. One wrote, "I love that we have the opportunity to focus on our understanding of math. We as teachers never get to practice our math skills. They model the problem-solving skills and learning techniques that we should be using in the classroom." According to Rodin, "We have been very impressed by the enthusiasm of the teachers and how they have responded to the Circle experience."

Links and Resources

MTC of Austin
http://sites.google.com/site/mtcaustin/home

MTC of Austin session handouts
http://sites.google.com/site/mtcaustin/home/problem-solving-sessions/previous-workshops

May 2011 meeting of the MTC of Austin.
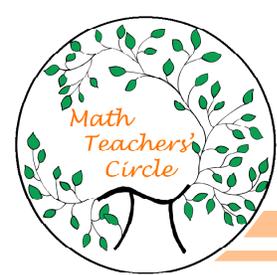
"Leavitt" to the Mad Vet: MTC Session Inspires Research Connection

"A Mad Veterinarian has three transmogrifying machines. Machine 1 turns one cat into two dogs and five mice. Machine 2 turns one dog into three cats and three mice. Machine 3 turns one mouse into one dog, one cat, and two mice. All the machines also work in reverse. Can the Mad Vet take one cat and turn it into seven mice?"

When Dr. Gene Abrams first heard this example of a so-called "Mad Veterinarian" scenario, presented by Joshua Zucker at a workshop on "How to Run a Math Teachers' Circle" held at AIM in 2008, something about the puzzles reminded him of Leavitt path algebras, his primary area of mathematical research. Abrams, a Professor of Mathematics at the University of Colorado Colorado Springs (UCCS) and leader of the Pikes Peak MTC, was so intrigued that he kept thinking about Mad Vet scenarios long after the workshop. According to Abrams, "It took me a few weeks to work out the connections, but when I did, it brought a big smile to my face. I've been involved in teacher professional development for about a decade, and this is the first time I've been able to relate content from a professional development workshop to my research."

The session at AIM focused on the mathematical idea of invariants. Participants figured out that there were only certain ways that the machines could transform any given configuration of animals, and so could determine when it was impossible to go from a given starting configuration to a particular ending configuration. Abrams developed this idea further by noticing that the collections of animals, or "menageries," can be divided into equivalence classes, such that each menagerie in a given equivalence class can be transmogrified into any of the other menageries in that equivalence class. The set of equivalence classes for a given Mad Vet scenario, together with the operation of addition that allows these equivalence classes to be combined, forms a semigroup. Abrams also realized that Mad Vet scenarios could be represented as directed graphs, and was able to use this graph theory representation to answer questions about the Mad Vet semigroups, such as when these semigroups are groups.

To describe this work and its deeper connection to Leavitt path algebras, Abrams co-authored an article with Ph.D. "sibling" Jessica K. Sklar (Pacific Lutheran University). Their article, entitled "The Graph Menagerie: Abstract Algebra and the Mad Veterinarian," appeared in the June 2010 issue of *Mathematics Magazine* (pp. 168-179), which is published by the Mathematical Association of America. "*Mathematics Magazine* turned out to be the perfect outlet for this work, which is accessible at an elementary level and yet contains connections to deeper research questions," said Abrams.

Abrams has since shared his work with various audiences, leading sessions of the Pikes Peak MTC and the Boulder Math Circle, giving departmental colloquia at UCCS and The Colorado College, and designing an activity for his undergraduate abstract algebra course. "It's a great topic that turns out to be accessible at many levels," said Abrams. "It's particularly fun and eye-opening to see how teach-

June 2010 cover of *Mathematics Magazine*, reprinted with permission from the Mathematical Association of America.

ers respond to and enjoy discovering mathematics."

Abrams and Sklar will be presented with the MAA's Carl B. Allendoerfer Award for their article at this year's MathFest meeting.

Links and Resources

Pikes Peak Math Teachers' Circle
http://www.uccs.edu/~pipes/ppmtc2.html

Mathematics Magazine, June 2010 issue
http://www.maa.org/pubs/mag_jun10_toc.html

"The Mad Veterinarian on Mathematical Safari," handout and slides
http://www.mathteacherscircle.org/resources/sessionmaterials.html
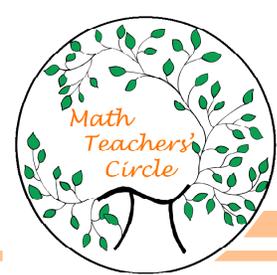
Featured Teacher: Susan Holtzapple

Susan Holtzapple

Susan Holtzapple teaches Algebra, Geometry, and math enrichment to 7th and 8th graders at Cupertino Middle School in Cupertino, CA, and also coaches her school's math competition team. She has been teaching for 11 years and joined the AIM MTC in 2008.

What inspired you to start participating in a MTC?

Within a 40-minute drive of my school, we have the San Francisco, San Jose, and Stanford Math Circles for students. Starting 5 or 6 years ago, I began encouraging my students to attend these Math Circles and would occasionally go myself to observe.

At one of the student Math Circle meetings I attended, Sam Vandervelde [one of the original founders of MTC] was the session leader. The kids had pretty much solved a problem, and then Sam asked them, "So what do mathematicians do now?" I was sitting there thinking, "I don't know, what do they do?" and then Sam responded, "They ask questions!" Suddenly all the kids were asking questions, and it was so cool. I thought, "I wish I could teach like this!"

What keeps you coming back?

I didn't really *do* math until I was 40. I was a science major and went to medical school, then spent time raising my family, before deciding to become a teacher. To be an effective teacher, it's necessary to be aware of how cool math can be and keep a real connection with math. MTC also helps remind us of what it's like to be a student, to be confused and to be learning. I think that every math teacher needs to do MTC! It's also a social highlight for me.

What has been your favorite session and why?

It's hard to pick out just one, but I really like the sessions where you end up finding the solution in a surprising way, like by using a different representation. I also like the probability problems, because they look impossible at first, but then when you break a problem down into smaller pieces and find you can solve it, it's a real confidence booster. I also like problems where you may have played with them before, but the session makes you realize that you barely scratched the surface. For example, we did a session on finding a fraction in between two other fractions, and I got totally lost, which was fun. I love being given a problem that I don't know how to do and solving it.

Do you feel that participating in MTC has influenced your teaching? If so, how?

My challenge is going from a concept I want to teach, to teaching it using problem solving. For the past 6 years, I've been thinking about how to do this with the required curriculum. To really make it work, I think you need to have exposure to lots of different problems, sometimes over and over, and a chance to play with them. I recently tried a problem-solving activity that I thought worked really well with my Algebra class. I had them look at what happens when you add strings of consecutive integers. For example, what happens if you add 2 consecutive integers? How about a string of 3, 4, or 5 consecutive integers? What patterns do you notice? What numbers can't you get? Why can't you get them? I also asked them, "What are all the strings of different lengths that make 255? How do you know you've gotten them all?" My students had a great time working together on this and I think learned a lot about noticing patterns and generalizing from them.

What do you like most about teaching?

I especially enjoy teaching 8th graders, because kids really start to understand humor at that age. I teach a lot of accelerated students, and it's fun watching them go from not feeling challenged when they first come to my classroom, to really blossoming. I love finding the edge of knowledge that kids don't have and challenging them to go further. I want them to understand what learning is—it's being confused! If they work on something for 20 minutes and still don't understand or are frustrated, that's okay—they're learning! I try to give them at least one thing during the year that they can't do, because I like showing them that there's so much more to learn.

Links and Resources

AIM Math Teachers' Circle
http://www.mathteacherscircle.org/circle/index.html
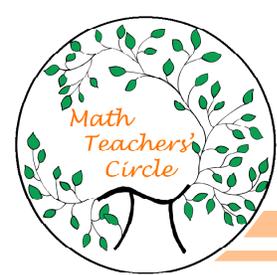
MTC Sessions: Grid Luck

by Joshua Zucker

Tatiana Shubin has a marvelous collection of problems entitled "Grid Luck" on the MTC Network website. These problems are thematically linked because they all can be solved more easily if you use graph (grid) paper rather than plain or lined paper. Any one of these problems could suffice for a lengthy MTC session! Several of them could also be easily adapted for use in a middle school classroom.

For example, Problem 9 asks students to cut a square into smaller squares. They will typically see right away that you can cut a square into 4, 9, or 16 squares, but it might take them a while to realize that the smaller squares don't all need to be equal in size. They then see how to add 3 (or 8) by cutting one square into smaller congruent squares. They may still think 6 squares is impossible, though, until they think a little differently! The resulting picture might help them remember that $(x+1)^2 = x^2 + 2x + 1$.

Problem 2, which asks how many grid squares are crossed by the diagonal of a rectangle of any given size, is one nice way of introducing a discussion of greatest common divisor. This same tool is essential in Problem 7:

> Draw a rectangle and trace the path of a billiard ball that begins in the lower left corner and initially travels upward at a 45-degree angle. Assuming that the ball bounces off the walls at perfect 45-degree angles, which corner does the ball reach first? What fraction of all the unit squares within your rectangle does the ball pass through on its way?

In working on this problem, students will uncover the utility of greatest common divisor and relate it to simplifying fractions, while possibly also discovering some interesting parity patterns. There's even some very subtle symmetry you can introduce with your graph paper, by reflecting the billiard table and letting the ball go straight instead of reflecting the ball's path. This turns out to be very illuminating!

I have worked on this problem with some sophisticated middle school students, and a brief outline of what we did is on the first page of my "Bouncing Ball Problem" handout and is illustrated by the diagrams here. There is also a long discussion of the problem in *Mathematics: A Human Endeavor* by Harold Jacobs, which you can read in Google books if you don't have your own copy.
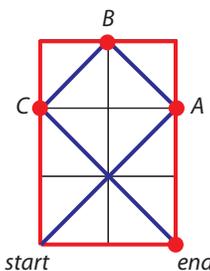
The ball can bounce, reflecting off the sides...
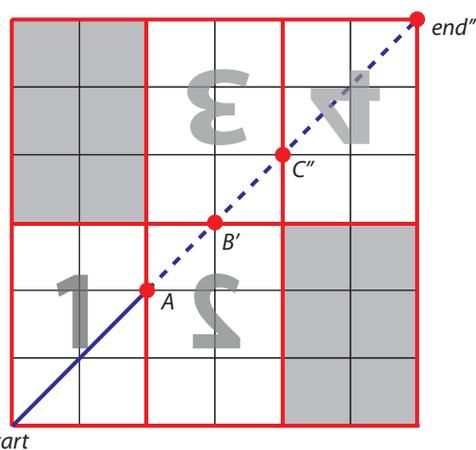
...or the sides can reflect while the ball goes straight!

MTC Calendar of Events

For more information about upcoming workshops by member circles please visit:
http://www.mathteacherscircle.org/workshops.html

June:

June 1-4, 2011
MTC of Hawai'i (MaTCH) Summer Retreat
University of Hawai'i, Manoa, Hawai'i

June 2-4, 2011
14th Annual Legacy of R. L. Moore Conference
Washington, D.C.
http://legacyrlmoore.org/events.html
The 14th Annual Legacy of R. L. Moore Conference will feature a plenary session on MTCs given by Judith Covington (LSU-Shreveport) and Angie Hodge (NDSU). There will also be a contributed paper session on MTCs and Inquiry-Based Learning.

June 6-8, 2011
Acadiana MTC Summer Retreat
Vermillion Conference Center, Lafayette, LA

June 13-24, 2011
MTC of North Dakota Summer Academy
North Dakota State University, Fargo, ND

June 20-24, 2011
University of Utah Teachers' Circle Summer Workshop
University of Utah, Salt Lake City, UT

June 22-24, 2011
Pikes Peak MTC Summer Academy
Mountain Thunder Lodge, Breckenridge, CO

June 26-30, 2011
New Mexico Summer Mathematics Circle Workshop (Albuquerque MTC)
Taos, NM

June 27-July 1, 2011
"How to Run a Math Teachers' Circle"
American Institute of Mathematics, Palo Alto, CA

July:

July 13, 2011
Casco Bay MTC Summer Workshop
University of Southern Maine, Portland, ME

July 18-21, 2011
Northern Louisiana MTC Summer Workshop
Louisiana State University-Shreveport, Shreveport, LA

July 18-22, 2011
Rocky Mountain MTC Summer Workshop
University of Colorado Denver, Denver, CO

July 18-22, 2011
"How to Run a Math Teachers' Circle"
MAA Carriage House, Washington, D.C.

July 25-29, 2011
New York MTC Summer Workshop
Bard College, Annandale-on-Hudson, NY

July 27-29, 2011
Mobile MTC Summer Workshop
University of South Alabama, Mobile, AL

August:

August 4, 2011
MTC of Austin Summer Workshop
University of Texas at Austin, Austin, TX

August 4-6, 2011
MathFest 2011
Lexington, KY
http://www.maa.org/mathfest/
The Special Interest Group of the MAA for Math Circles for Students and Teachers (SIGMAA MCST) will host a contributed paper session on the morning of Friday, August 5. Other events including Circle demos are planned.

Details of upcoming events, including MTC meetings and workshops and conferences of interest to the MTC community, may be sent to circles@aimath.org for inclusion in the *MTCircular* calendar.

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- Uri Treisman, *UT Austin*

If you would like to nominate a teacher for our featured teacher segment or would like to suggest a story for *MTCircular* please send an email to circles@aimath.org.

To see a list of MTC Session Materials featured in this issue, please visit http://www.mathteacher-scircle.org/resources/sessionmaterials.html.

MTCircular is edited by Brianna Donaldson and designed by Grace Mathieson.
To subscribe, please send a message to circles@aimath.org.