MTCircular

Autumn 2012

Be Less Helpful

EMBRACING PERPLEXITY FOR A PRODUCTIVE CLASSROOM

+

Math Teachers' Circle π

Less Helpful, More Successful

Successful MTC session leaders must serve as guides, keeping in check the nearly irresistible urge to tell the participants about all the wonderful mathematics their topic leads to, while at the same time subtly directing the participants toward the information and tools they need to explore and discover that mathematics for themselves. In this issue, Joshua Zucker, a founder of the MTC Network and one of the primary developers of the MTC session style, shares his thoughts on this balancing act and interviews other expert MTC session leaders Tatiana Shubin, James Tanton, and Paul Zeitz, as well as Alan Schoenfeld, an expert on teaching problem solving, about their own approaches to "being less helpful."

I was struck by how all of the articles in this issue of MTCircular come back to this same theme. Clinton Morrison's fascinating study of teacher networks points out the immediate and long-term benefits of teacher-driven discovery rather than "delivered" knowledge. Mary Belisle, a leader of the featured Casco Bay MTC, observes, "As a session leader, I've found that the meeting really only reaches its full potential when you really learn to back up and relax." Our featured teacher, Horda Semdani of the North Louisiana MTC, says something similar about her middle school classroom: "I have become more of a facilitator and less of a lecturer. I talk less, and, amazingly enough, they learn more!"

As our network continues to grow, I am continually impressed by the achievements of each of our Member Circles and the impact they are having on their communities. In the past year alone, Member Circles have collectively raised over $350,000 for their local meetings and workshops. This issue's "News and Views" column highlights prestigious teaching awards and competitive grants won by Circles and Circle leaders since our last newsletter. Keep up the great work!

Happy problem solving!

Brianna Donaldson

Brianna Donaldson, Director of Special Projects

American Institute of Mathematics

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by Joshua Zucker

Be Less

Photo: iStockPhoto

A butterfly that has just completed its metamorphosis must struggle very hard to emerge from its chrysalis. And there is an important reason for this natural struggle. Fighting to get out is a key exercise that strengthens the butterfly's new wings enough to fly. In fact, if someone were to attempt help the butterfly out of its cocoon, bypassing the uncomfortable but essential struggle, its wings would remain soft and weak and it would never fly. Similarly, as teachers, we all want to help our students, but, at times, being too helpful is precisely what hurts them most. Here's why we must try to

Helpful

Photo: Chris Watk, stock.xchng

At their worst, too-helpful teachers replace problem solving with guessing. Students offer some possible answers and then, once the teacher tells them which one is correct, the students write that down and go on to the next problem. They're learning that having the right answer is what matters, and becoming dependent on the teacher to tell them when they have it.

It can even be too helpful when the teacher offers a suggestion like, "Try the distributive property." The teacher feels helpful, the student feels helped, and everyone feels happy, until the next problem comes, and the next, and the student still doesn't know how to start without the teacher pointing in the right direction. "This is much easier to avoid when you can escape the culture of 'how' and 'what' questions and move toward 'why' and 'what if,'" research mathematician James Tanton said.

Of course, at the other extreme, there are teachers who simply lecture and then leave the students to do the work all on their own, without any help. These teachers probably need to learn how to be more helpful. But then, that sort of teacher probably isn't reading this newsletter! For most of us, the much bigger danger is giving in to the temptation to be more helpful rather than making the students think as much as possible for themselves.

The Search for Perplexity

Stanford University's Dan Meyer has a well-known blog, "dy/dan," with the motto, "less helpful." What does this mean, and why is it a good thing?

To be less helpful sounds like bad advice on the surface, and maybe that's why it turns out to actually be good advice: it makes you stop and think. Dan Meyer points in the direction of "perplexity," which is the feeling of puzzlement that makes you want to work something out (as opposed to confusion, which is when you feel like you have no tools at all to figure out what to do next). We want to be less helpful, not unhelpful, so it's important to give students the strategies to deal with their situation while making sure that they have real work to do. For example, rather than telling them, "Try the distributive property," we could ask them, "What form would you like this expression to have?" and make a connection to the very general strategy of wishful thinking.

To be the least helpful of all, we can indeed leave the student or the Circle to solve the problems, but we can also have them create the questions. "It takes only a few minutes to let them form their own questions," says Tatiana Shubin of San Jose State University. "This is an area where mathematicians can really contribute to a MTC. When circlers form their own questions, mathematicians are in a position to see whether they are leading toward something deep and interesting."

A Productive Classroom

Alan Schoenfeld of UC Berkeley calls a classroom in which the teacher is not too helpful a "productive classroom," and points out that being less helpful means placing more of a cognitive demand on the students. Students may fight back if they're not accustomed to this, and press the teacher toward being more helpful, and possibly too helpful, but it's quite clear that this natural reaction leads to less learning. Schoenfeld has done substantial work on this question in collaboration with Bob Floden at Michigan State University and others and ended up with five key questions and a rubric to evaluate how successful the classroom environment is in producing mathematical thinking by the students.

- **Mathematical content:** Is it all skills and procedures, or is there significant attention to concepts and connections? This ties in well with the mathematical practice standards of the Common Core State Standards.
- **Cognitive demand:** Is there "productive struggle" or "perplexity" on the part of the students, or are things broken down into steps so small that the students never have the opportunity to make an effort?
- **Access:** Is everyone involved, or could only a few students participate while others spectate?
- **Agency:** Is the teacher the authority, the source of information, and the judge of quality, or are students also involved in explaining their thinking and responding to other students' ideas?
- **Assessment:** Is there continuous use of student reasoning to advance the discussion and sometimes alter its path, or is the feedback limited to a correction when there is an error? Or, even worse, is student reasoning not elicited at all?

In the worst case, classrooms fail to meet any of these goals. Paul Zeitz at the University of San Francisco calls this the "foie gras method." "That's when we stuff the subject with material and then test how well they can excrete it at a scheduled time," Zeitz said. "We're trying to avoid the stuffing and regurgitating. We want to teach people to be brave, to engage with hard questions, and to be willing to flail around." In Zeitz's book, "The Art and Craft of Problem Solving," one section leads with the epigram "The explorer is the person who is lost." Zeitz explains, "We want to teach people to be happy to be lost."

Creating Mathematics For Themselves

Math Teachers' Circles help us learn to enjoy being lost. Our sessions focus on mathematical content, concepts, and connections. We emphasize perplexity, giving problems that are challenging enough to eliminate boredom and accessible enough to avoid frustration. We wait, giving time to think, so that every participant can get involved and they can respond to each other. "The participants need to feel like they create the mathematics for themselves," Tanton said. "We set the stage, and while we may enjoy performing, it has to be their experience." We come prepared with more material than we could possibly use in a session so that we can move flexibly in the direction that the participants choose. We show our love for the mathematics, but for participants to fully appreciate the beauty of it, they must find it for themselves.

We've always known that a problem-solving focus with rich mathematical content does us no good if it's the mathematician or teacher doing all the solving with the participants as spectators. In MTCs, we refer to "session leaders" rather than "presenters" or "lecturers," to make the role more clear. We emphasize that they should be sure to give the participants time to think. Now we can add a reminder for session leaders: Be less helpful! Create a cognitively demanding classroom! Allow your students to fight their own way out! It is only then that they will be able to fly.

The author wishes to thank Sally Bennett for the original idea of this article and Emily McCullogh for her helpful comments regarding this article.

Tips to Remember

- **Give students/participants enough time to think and deduce.** Don't give away too much too soon.
- **Let participants generate the questions.** This creates engagement in, and ownership of, the investigations that follow.
- **Encourage perplexity.** Give problems that are challenging enough to eliminate boredom, yet accessible enough to avoid frustration.
- **Be prepared to only get through a fraction of your material.** Let the participants' interests and questions dictate the direction the session goes.
- **Focus on more general strategies.** Give nonspecific hints rather than giving answers away.

Links and Resources

dy/dan. Dan Meyer's mathematics blog.

Floden-Schoenfeld Algebra Teaching Study.

The Art and Craft of Problem Solving. Paul Zeitz.

For links to these resources and more, visit us online at ***http://mathteacherscircle.org/resources/sessionmaterials.html****.*

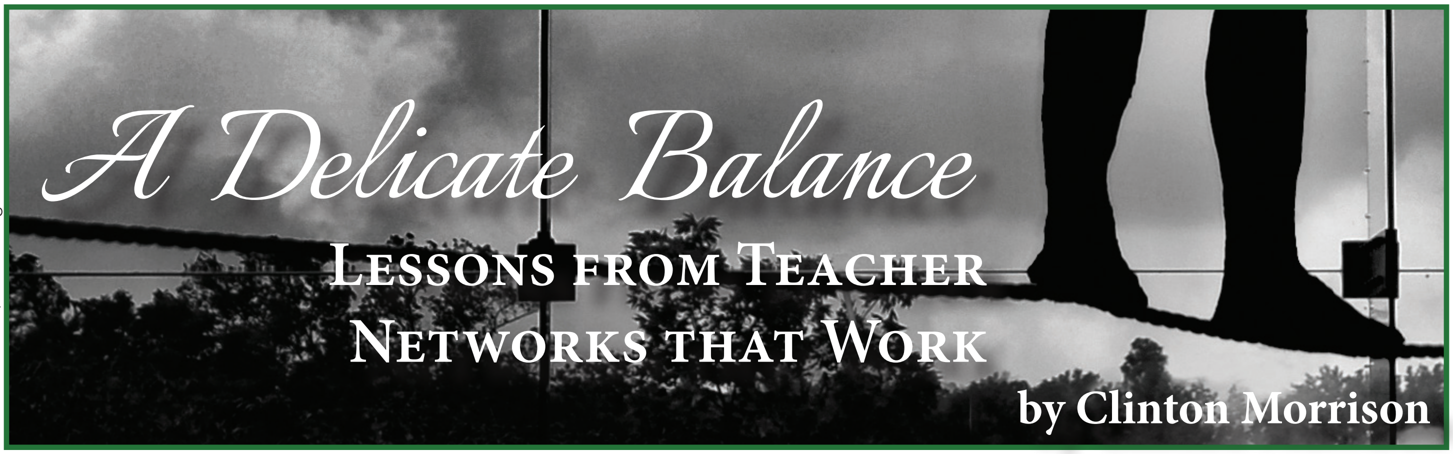


Photo: Kristin Smith, stock.xchng

A Delicate Balance

Lessons from Teacher Networks that Work

by Clinton Morrison

Teacher networks have been recognized in recent years as a bright spot in the vast and somewhat confusing discourse on school reform in the U.S. Why? Because these networks work. They transform a teacher's practice by immersing them in a pool of invaluable knowledge and collective experience. Teachers then bring their newfound confidence and expertise into the classroom. As it turns out, keeping such a network running smoothly requires maintaining a delicate balance between various tensions and needs of the group. These tensions include:

- **Learning vs. Collegiality:** The emphasis placed on member learning vs. member camaraderie, or a learning environment vs. a social environment.
- **Content vs. Pedagogical Knowledge:** The emphasis placed on concrete knowledge and concepts vs. reflection on the craft of teaching itself.
- **Delivered vs. Constructed Knowledge:** The emphasis placed on knowledge that is prescribed and injected by established network leaders without member input vs. letting network interfacing to take its own course, allowing for self-reflection, discovery and equal leadership opportunities.
- **Outside vs. Inside Knowledge:** The emphasis placed on contribution of visiting experts vs. collective peer knowledge within the network.

What stance does a successful network take in these tension areas and what specific strategies does it use to reconcile them? Leaders of four exemplary, well-functioning, sustainable teacher networks, including the nationwide Math Teachers' Circle Network, weighed in with tips on how they keep their own networks ticking.

Learning vs. Collegiality

Research shows that networks that focus on relationships among teachers first are more successful than ones that neglect this aspect. In order to establish a sustainable foundation among members, time and financial support should be devoted to activities that build trust and a common identity.

Above all else, foster relationships first; trust and bonding are the foundations of learning and longevity. There are a number of ways to solidify relationships in a network, but to neglect the process altogether is fatal. Many MTCs overcome this challenge with a strong emphasis on the social aspects of the Circle. Friendships are forged in intensive founding workshops and the social aspects of the Circle are further emphasized with ongoing Internet contact, end-of-year social events, and often, a shared meal to start evening meetings throughout the year. Initially, the relationship-building process is important because it builds the trust necessary to forge a path to learning and participation. Second, positive relationships keep teachers motivated to continue to interact. Teachers are more willing to incur the burdens of travel if they are excited to see their colleagues, who have become good friends. Third, strong relationships perpetuate network energy and foster growth.

Find ways of incorporating relationship-building into the learning agenda as seamlessly as possible. The easiest way to get anyone to learn is to make it fun and interactive. The MTC Network employs a team-approach philosophy of problem solving in its workshops to simultaneously foster friendships and learning.

Content vs. Pedagogical Knowledge

Networks struggle to find reconciliation regarding the topics they choose to emphasize. Content knowledge includes curriculum and subject knowledge; procedural knowledge, such as lesson plans and classroom management; and discrete technical knowledge. Alternatively, pedagogical knowledge involves the broader questions of teaching, demanding self-reflection and abstract conceptual inquiries about teacher-student roles.

Use demographic knowledge of the network population to target the emphasis of knowledge type. Representatives from each of the teaching networks surveyed observed that, of each general area of knowledge, it is content that teachers lack the most. And indeed, if teachers are only one step ahead of their students in content knowledge, then they cannot help them to make the broader connections necessary for a rigorous education. All of the networks initially went through some process of research gathering (generally informal) to determine the knowledge void they were intended to fill.

Build pedagogical expertise into the context of teacher learning. Here, the MTC method serves as an outstanding example of how a network might successfully reconcile this area of tension. The focus of the learning is on content, but the social structure within which teachers interact both tacitly and explicitly challenges them to reconsider the teaching craft. As teachers struggle collectively to formulate solutions to rich math problems, they find themselves engaged in a style of learning that inevitably compels them to reconsider their own classroom model. Teachers often contemplate, "I'm absorbing so much information and it's fun too. Could my classroom work this way?"

Delivered vs. Constructed Knowledge

This tension area primarily examines knowledge sources. Delivered knowledge in its pure sense is that which is prescribed, preplanned and presented to network participants with little to no reciprocity, as in a lecture. Constructed knowledge is defined by egalitarianism and spontaneity; network members learn and interact without clear premeditated direction and leadership.

Build supports, not obstacles; ultimately, structures should be teacher-driven. The only way to know how to successfully balance delivery and construction is by clearly understanding what teachers want. Pursuant to this, all networks evaluated carry out all of their core activities with a high level of teacher input and provide clear and easy avenues for feedback.

Reward progress and exhibit accomplishments to empower learners. Many teachers begin their network participation preferring to be learners; however, great networks change individual perception over time by engendering transformation. The transformative experience occurs when novice teachers deeply invest themselves in the network and in turn find at some point that they have progressed greatly and are confident in being a leader in the network and in their schools.

Outside vs. Inside Knowledge

A network that utilizes outside knowledge brings in "experts" or field practitioners not inculcated into the network to contribute in a variety of ways. Inside knowledge constitutes all knowledge and experience of network members. The search for this delicate balance poses the question, "How do you improve capacity without implying incapacity?"

Always ensure that teachers feel in control of their learning. Networks that successfully balance the use of outside expert knowledge emphasize the view that all network activity should be teacher-driven. The MTC Network employs a paradigm where an expert mathematician will begin a discussion on a topic, often with a brief presentation, followed by at least an hour of teacher-driven open conversation. In such cases, teachers welcome the inclusion of outside knowledge as it is used as a tool simply to inform a broader dialogue on the topic.

Utilize referrals to specifically target experts with teaching skills. Because the MTC learning model is built around the mathematician-as-leader idea, it is important to integrate the expertise of leaders with some level of teaching experience or, at minimum, strong communication skills. Many MTCs work around this problem by zeroing in on prospective session leaders with comprehensive skills in math and teaching, or, alternately, having a mathematician co-lead a session together with a teacher.

Clinton Morrison is a Master of Public Administration candidate at the University of Colorado Denver. This article contains research he completed for the Colorado Legacy Foundation to help launch a statewide network for Advanced Placement teachers. ***View a copy of the complete study on our website****.*

FEATURED CIRCLE

Five Cooks in the Kitchen

How the Casco Bay MTC Makes it Work with a Five-Person Leadership Team

It's a Saturday morning in Maine, but five particularly dedicated teachers are heading to work on their day off. Some of them will drive more than an hour to get to Portland, where they will all crowd into one room for a video conference call interview. If you want to speak to the leader of the Casco Bay Math Teachers' Circle, you'll have to wait until all five of them are available. Because all five of them are equally ranked leaders of the MTC, and they wouldn't want it any other way. Here, the leadership team, consisting of Muhammad El-Taha and Peggy Moore of the University of Southern Maine and Mary Belisle, Sally Bennett and Shawn Towle, three Maine middle school teachers, share the story of their group and offer their advice to other Circles.

How did your group get started?

Muhammad El-Taha: As a college professor, when I became the chair of my department at the university, I became very involved in math classes of every level, instead of just the classes I personally taught. I was taken aback by the students' severe lack of knowledge in certain areas, particularly in remedial courses. I asked the teachers, "What happened here?" They blamed it on the high schools. I asked some high school teachers, "What happened here?" and they blamed it on the middle schools. When I arrived at the middle schools, they did not point me toward the elementary schools, so I figured I had found the weak link. So when I heard about the Math Teachers' Circles and how they were especially concerned with the formation of middle school math teachers, I knew I had found the right place. Then all five of us attended the summer 2010 AIM workshop in Washington, D.C.

Sally Bennett: And in fact, some of us met in person for the first time at the workshop. But we really bonded at that workshop and that's really what helped get our Circle started. We kicked off our group in the spring of 2011 and it's been going strong ever since.

What is the secret of your Circle's success?

Shawn Towle: It actually wasn't that overwhelming to get the group off the ground. Using the resources from AIM helped a lot. They really make it so you don't have to reinvent the wheel.

Photos courtesy of Casco Bay MTC

Left, Peggy Moore checks on a group of working teachers during a MTC session. Right, Muhammad El-Taha offers a hint.

Peggy Moore: We often lead the sessions as pairs, and that also takes a lot of the stress off. You can bounce off each other, like a relay.
Bennett: We also move people around during the meeting, so they don't always sit in the same place. This is a highlight for our members, to make new friends and see new people.
Mary Belisle: Also, the schedule is set months ahead of time, and the meetings are always on the same day of the month, the first Wednesday. That way it's not a moving target and people are really able to plan ahead and make it a priority.

What are some of the biggest challenges your MTC faces?

Bennett: We currently have a problem that many Circles face, and that is a lack of funding.
El-Taha: There is also a problem of time. Any time you try to get five people together to do one thing, you are going to run into problems. Also, we don't all live very close by. It is a real challenge to stay connected. But luckily, we are all very heavily invested in this group, so it is less of a chore than it could be.
Moore: Plus, we really enjoy each other's company. That makes a big difference. I don't think we would be nearly as passionate about the Circle if we didn't enjoy working together so much.

Do you have any advice for someone who is gearing up to lead an MTC session?

Belisle: As a session leader, I've found that the meeting really only reaches its full potential when you really learn to back up and relax.
Moore: Sometimes in meetings, the session leader doesn't even know quite where the train is going. Getting used to that feeling can be difficult, but just coming along for the ride can be really exciting too.
Bennett: When choosing a problem, I look for something that I personally don't understand. A recent topic that really provoked interesting discussion was James Tanton's exploding dots.
Towle: We recently had another very lively session concerning handshakes and how it relates to graph theory. There was also a great session on domino tiling, where the teachers worked in groups and came up with different solutions that were all correct.
El-Taha: We also enjoy coming prepared with different problems that are connected by a common mathematical theme. Then the group can make connections that even the session leader didn't see at first.

What is the best thing about your MTC?

El-Taha: It's given me a new appreciation for the middle school teachers.
Bennett: It's great to meet with a group of people who choose to do math for fun.
Moore: It's priceless to watch a roomful of adults playing with dominoes, not because they're being paid for it, but for the sheer joy of learning. Plus, I feel that the five of us really work well together and we have a great group dynamic going.
Bennett: And it is our hope, at least, that the teachers involved in the Math Teachers' Circle leave with the tools to better prepare their students for the future. And I truly think it's working so far.

Links and Resources

Casco Bay MTC home page

James Tanton's Exploding Dots.

Handshakes and Graph Theory. E. Johnston.

Trio of Friendly Problems. Classroom lesson plan regarding handshakes and more, M. Fay-Zenk.

Dominoes and Rectangles and Tiling Torment, J. Zucker.

For links to these resources and more, visit us online at http://mathteacherscircle.org/resources/sessionmaterials.html.

Poetry and Proportions

Five Questions for Horda Semdani of the North Louisiana MTC

When Horda Semdani came to Louisiana from Belgium in 1993, she started out teaching 8th grade French and English. But she knew that teaching math was her true calling, and because she enjoyed it so much, she spent a few years tiding herself over by tutoring math for free in her spare time. Now, six years into her math-teaching career, she finds herself teaching various levels of math to 7th graders at Greenacres Middle School in Bossier City, Louisiana, in the greater Shreveport area. Last July, she began attending her local MTC, and she says it has changed the way she'll view math teaching forever.

What caused your initial interest in Math Teachers' Circles?

My supervisor told me about our local Circle, the North Louisiana MTC, last summer. She mentioned that the teachers in the group were really engaged and that their students were starting to follow suit. Engaging my students is always a high priority for me, especially with my female students. It seems that girls always think that math is too hard or too nerdy and they refuse to get interested. I thought I could maybe learn something from the MTC on how to get them interested. Luckily, it has paid off in the best possible way. Not only are the girls in my classes more interested now, but so are the boys and so am I.

How has your experience with your MTC changed your teaching style?

The best thing I have learned so far is the concept of problem posing vs. problem solving. In problem solving you, as the teacher, already know the answer, whereas in problem posing, you're just along for the ride. It's the difference between endless boring worksheets and a lively discussion where everybody learns something. For the most part, the concepts we cover in MTC sessions tend to focus on project-based learning and figuring out concepts on our own. People learn better when they figure it out rather than just being told what to do. The really interesting thing is that, with this teaching style, I have become more of a facilitator and less of a lecturer. Instead of jumping through hoops to teach the kids something, I just get a discussion going and they take off with it. In fact, I talk less, and, amazingly enough, they learn more!

What is an example of a topic that has worked well in your classroom?

Advocating MTCs to the Public

Horda Semdani and her Math Teachers' Circle, the North Louisiana MTC, together with Circle leader Judith Covington, were recently featured in an "Assignment: Education" segment for Shreveport news station KTBS 3. "I have seen my kids going from being just well managed to highly engaged," Semdani told the news station. **View the segment**.

Photo courtesy of Horda Semdani


Semdani, third from left, poses with some of her 7th graders.

I am a firm believer in cross-curricular learning, that is, making connections between subjects. So learning math through music or poetry, for instance, helps kids with different learning styles make a connection, as well adding general interest. One great example is a recent lesson in which I read the poem "One Inch Tall," by Shel Silverstein, aloud to my students. The poem talks about how different life would be if the author were only one inch tall. Then the class had a long discussion about proportional relationships, using math to figure out how other things would be different if they were only one inch tall. The kids were learning a required 7th grade concept from the Common Core State Standards and they had no idea! They just thought it was a fun, and slightly silly, discussion.

What is your favorite thing about teaching math?

It would have to be my students. I really respect my kids and value their opinion. I have also found that my enthusiasm for what I do really makes a difference. I dress up in a suit to teach every day, and the kids take notice. I really want to make a difference and change their lives for the better, and when a kid who hates math learns to love it, or when one of my students moves up from a regulars class to an honors class, I celebrate with them.

Do you have any advice for other teachers?

If you give kids some choices, it makes them feel like they're more in control. That can do great things for their self-esteem, and it makes them far more interested in learning. There are great things to be learned at a Math Teachers' Circle, and it can be very rewarding to try to translate it into something that will speak to your students. Sometimes changing your teaching style can feel a bit overwhelming, but try to be flexible and you'll be amazed what they, and you, can learn. You'll be more productive, and your kids will be more successful. And everyone will have a lot more fun learning in the process.

Links and Resources

Contact **Judith Covington**, leader of the North Louisiana MTC.

Math Doesn't Suck by Danica McKellar. Math resources and encouragement for girls.

"**One Inch Tall**." Poem by Shel Silverstein.

Local news segment featuring Semdani discussing benefits of MTC membership.

For links to these resources and more, visit us online at ***http://mathteacherscircle.org/resources/sessionmaterials.html***.

North Louisiana MTC Leader Judith Covington Receives Distinguished Teaching Award

Congratulations to Dr. Judith Covington, Associate Professor of Mathematics at Louisiana State University Shreveport and director of the North Louisiana MTC, who has been awarded the Distinguished Teaching Award of the Louisiana-Mississippi Section of the Mathematical Association of America. Covington was also recently recognized with the "Champions for Mathematics Education" award from the Northwest Louisiana Mathematics Association. The North Louisiana MTC was also recently featured on local ABC-affiliate KTBS 3. For more information on the news segment, see page 12.

Jason Ermer of MTC of Austin Honored With Teaching Excellence Award

Congratulations to Dr. Jason Ermer, recipient of the College of Natural Sciences Teaching Excellence Award from the University of Texas at Austin. Ermer, a Clinical Assistant Professor in the UTeach program, is also a founding member of the MTC of Austin. In his work at UT Austin, Ermer facilitates field experiences for pre-service secondary STEM teachers in the UTeach program and regularly teaches mathematics content courses for pre-service elementary teachers. Ermer's instructional style is to teach mathematics through problem solving and in his work with the MTC of Austin, he helps in-service teachers develop their own problem-solving skills.

Northern Virginia MTC Leader Bob Sachs Wins Teaching Awards from George Mason, MAA

Congratulations to Dr. Robert Sachs, founding member of the Northern Virginia MTC and recent recipient of the David J. King Teaching Award from George Mason University. The David J. King Teaching Award is given annually to a faculty member who has made significant, long-term contributions to the overall educational excellence of the university. In Spring 2011, Sachs was also the recipient of the John Smith Award for Distinguished College or University Teaching by the Mathematical Association of America in the Maryland-D.C.-Virginia section. Sachs is the first Mason professor to receive the award since its first recipient in 1992, the late John Smith, for whom the award is named.

Charlotte MTC Leader Harold Reiter Receives Nomination for Public Service Award

Dr. Harold Reiter, a founding member of the national MTC Network's leadership team, has been nominated for the University of North Carolina system's Award for Excellence in Public Service. The award was established in 2007 to "encourage, identify, recognize and reward public service by faculty of the University." Each of the constituent institutions may nominate one faculty member. This year UNC Charlotte is nominating Harold Reiter in recognition of his sustained public service, including his involvement in the MTC Network. Reiter has been involved with the MTC Network since its inception and has played an important role in the development of the successful national initiative. He attended the very first MTC workshop, held at AIM in 2006, and began efforts that same year to found a MTC for middle school teachers in the Charlotte area. The Charlotte Teachers' Circle was the first MTC to be formed outside the San Francisco Bay Area, reflecting Harold's commitment to bringing this innovation to the teachers of North Carolina. The winner of the award will be announced this fall.

Math Teachers' Circles Receive Grant Funding

Congratulations to the following MTCs that recently received grants from a variety of sources nationwide.

Seed Grants Through Private Donation to AIM

- $1,500 to Chippewa Valley (Eau Claire, WI)
- $1,500 to Eastern Kentucky
- $1,500 to Fairfield County (CT)
- $1,500 to New Hampshire
- $1,500 to Northern Colorado
- $1,500 to Omaha (NE)
- $1,500 to Philadelphia Area (PA)
- $1,500 to Roaring Fork Valley (Aspen/Carbondale, CO)
- $1,500 to Sioux Falls (SD)
- $1,500 to Winston-Salem (NC)

Seed Grants From MSRI/NAMC

- $1,100 to Chippewa Valley (WI)
- $2,000 to Eastern Kentucky
- $2,000 to Northern Colorado
- $2,500 to Fairfield County (CT)
- $2,500 to Philadelphia Area (PA)

Selected Other Grants

- $600 from Ann Marie Foundation to Chippewa Valley (WI)
- $2,400 from Xcel Energy to Chippewa Valley (WI)
- $5,000 from Aspen School District to Roaring Fork Valley (Aspen/Carbondale, CO)
- $14,000 from Verizon to Philadelphia Area (PA)
- $15,000 from Mikkelson Foundation to Roaring Fork Valley (Aspen/Carbondale, CO)
- $15,000 from Winston-Salem Community Foundation to Winston-Salem (NC)
- $20,000 from State Farm to Northern Colorado
- $72,550 from ESEA Title IIA, Improving Teacher Quality Higher Education Grants to MaTCH (HI)
- $90,000 from ESEA Title IIA, Improving Teacher Quality Higher Education Grants to South Carolina High Energy Math Teachers' Circle (SCHEMaTC)

NEWS AND VIEWS

Project Led by Christina Eubanks-Turner of Lafayette, Louisiana, Awarded NSF Grant

The National Science Foundation has awarded $127,449 to support a research project led by Dr. Christina Eubanks-Turner, leader of the Acadiana MTC in Lafayette, Louisiana, together with fellow primary investigators Dr. Peter Sheppard, Dr. Patricia Beaulieu and Dr. Kathleen Lopez. The Louisiana Noyce Teaching Fellows/Master Teaching Fellows Planning Project aims to develop the integration and implementation of a teacher preparation program for middle school teachers in Louisiana. "This grant is important because it is an effort to build capacity and infrastructure of STEM teacher preparation programs in Louisiana," said Eubanks-Turner. "Our planning project will enhance critical partnerships across disciplines and organizations that will lead to transformative change in STEM teacher preparation in Louisiana." According to the NSF, the University of Louisiana at Lafayette mathematics and curriculum/instruction departments will partner with school districts in Lafayette and Iberia Parishes and the Allen J. Celestine Foundation to develop the program. During the 12-month planning period, the team will identify best practices of successful teacher preparation programs, identify partnering school districts and nonprofit organizations, secure matching funds and develop a comprehensive curriculum for the program.

got news? events?

Keep the rest of the MTC Network up-to-date with events and news in your own Circle. It's as simple as submitting news items and notice of your upcoming events to

circles@aimath.org

News may be published on our website or in a future issue of MTCircular.

Events will be included on our online calendar at

http://www.mathteacherscircle.org/calendar.html

Problem Circle

Number Sum Triangles

This beautiful problem comes to us via the 2012 Bay Area Math Olympiad for Teachers and a Russian teacher olympiad, but it almost certainly has a long history before that. It is simple to explain – even a first grader can get started on it – but also surprisingly deep and full of unexpected connections. For example, what does it have to do with fractions and proving that the rationals are countable? How could it possibly relate to place value in base 2? In what ways is the pattern here like a fractal? We'll answer at least some of those questions in the solutions presented in the next issue.

Start with a list of the two numbers: 1, 1. Then, from that list, generate a new list by inserting the sum of each pair of numbers between those two numbers. Continue to generate new lists in this way. Thus, the next lists are 1, 2, 1 and 1, 3, 2, 3, 1 and so on. How many 2012s are found in the 2012th list?

1 1

1 2 1

1 3 2 3 1

1 4 3 5 2 5 3 4 1

Last issue's problem, "Number Bases," encouraged us to think in base -4 for the purpose of counting, adding, multiplying and writing decimals. Please visit the **Problem Circle page** on our website for a partial solution to that problem. Send your answers (or your MTC's answers) to this issue's problem to Joshua Zucker by **e-mail** to be featured in a future issue of MTCircular.

$$132 = 1 \cdot 16 + 3 \cdot -4 + 2 \cdot 1 = 6$$

Links and Resources

Bay Area Math Olympiad for Teachers.

For links to this resource and more, visit us online at ***http://mathteachercircle.org/resources/sessionmaterials.html****.*

In Session

Compositions

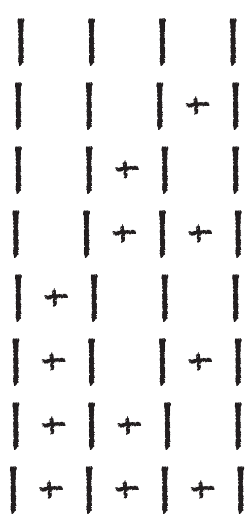
Partitions are the ways of breaking a positive integer into a sum of one or more positive integer parts, where we only care about the size of the parts and not their order. For example, there are three partitions of the number 3: 1+1+1, 1+2, and 3 itself.

Compositions are similar to partitions, but order matters. So, there are four compositions of 3: 1+1+1, 1+2, 2+1, and 3 itself.

Compositions are appealing and challenging enough to make a good MTC session, yet they're also accessible and useful in a middle school classroom. Middle schoolers can use them for simple single-digit addition practice, as well as some fundamental problem-solving skills. Dealing with compositions of large numbers requires both patience and organization, but also has great rewards like discovering a natural occurrence of powers of 2, Pascal's triangle, and the relationship between them. It can also be a great lesson in exponential growth. Finally, when students develop different ways of organizing their work, the results can lead them to even more complex patterns to explore further.

In leading an MTC session on compositions, I like to simply introduce the definition and leave it to the participants to start generating the questions. Probably the most natural question is to find how many compositions each number has. Teachers are also likely to start exploring how to bend the rules: What if order doesn't matter? What if you're only allowed to use each number at most once? What if you're only allowed to use odd numbers? All of these questions are good ones, but probably beyond the scope of what you can explore in one session.

So, let's say that you want to concentrate on how to count the compositions of a number. Multiple representations are key here. One particularly useful representation is "base caveman," using tally marks to write your numbers instead of base 10



Caveman Representation of Compositions of 4

1	= 1
1 1	= 2
1 2 1	= 4
1 3 3 1	= 8
1 4 6 4 1	=16
1 5 10 10 5 1	=32

Pascal's Triangle

numerals. When you write the partitions of 3 as follows: | + | + |, | + | |, | | + |, and | | |, you can see that you have a choice of inserting a + sign or not in the space between each tally mark. There are 2 such places, and hence $2^2 = 4$ possible compositions. And, in general, n will have 2^{n-1} compositions. (The representation with Ferrers diagrams is much more useful for partitions than compositions, but it might make a good aside if you have more time.)

That's one way to count them, but it might give away too much, too soon, to start there. So, instead, you might begin by suggesting that the teachers think of ways to organize the compositions for each number into groups to make them easier to count. Let them brainstorm a few possible organizational schemes and then try to work through them from simplest to most complex so they can discover new things as they go.

One way to organize compositions of a number is by the size of the last number listed in each composition (or by the size of the first number listed, which amounts to the same thing). After teachers have had some time to work, they'll be telling you about powers of 2, and if you're really lucky, they will explain that the number of compositions of 4 ending in 1 is the same as the number of compositions of 3 in total, because you append a +1 to the end of each.

A second way to organize the compositions of a number is by length. After some experimenting with this, teachers will notice Pascal's triangle! A good general approach for understanding why Pascal's triangle appears is to try to explain why each entry in the next row is the sum of two entries in the previous row. Why is the number of compositions of, say, 6 into 4 parts, the same as the sum of the number of compositions of 5 into 3 parts and of 5 into 4 parts? Making a list and staring at it for a while, you'll see that the compositions of 5 into 3 parts need a +1 at the end. But what about the ones that already have 4 parts? You still need to add 1 somewhere, but now you do it by increasing the last number by 1. This might be a good time to introduce the tally-marks representation: you can put + | at the end, or just put | at the end. It's clear enough that those are the only two ways you can add one more tally mark to a composition!

Another way to explain Pascal's triangle here also comes from the tally mark representation: To make 6 with 4 parts, you have 5 spaces and you have to choose three of them to put + signs into.

There are other ways to organize the compositions as well, that may seem more natural but turn out to lead to some more complicated counting results. For instance, you could organize them by largest part. You'll find some relationships to Fibonacci numbers there, as well as some more complicated things! You can also explore the diagonals in this organization scheme, which are the number of compositions of n with $n - k$ as the largest number, for some fixed k. You will see that these eventually become constant, but why? How far down the diagonal do you have to go to find the constant? And how can you predict the value of that constant in terms of k?

Links and Resources

Joshua Zucker, "Compositions."

For links to this resource and more, visit us online at ***http://mathteacherscircle.org/resources/sessionmaterials.html***.

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