The Teacher's Circle, August 14 -18, 2006

My name is Paul Zeitz, and I will be leading sessions on Number Theory, Probability, and possibly Geometry. These are seemingly unrelated topics, but all of mathematics is deeply interconnected, as you will see.

I grew up in New York City, went to a high school where it was socially acceptable to be a math nerd, and was a member of the first American team to compete in the International Math Olympiad (in 1974). I received a PhD in Math from UC Berkeley in 1992 and have been a professor at the University of San Francisco since then. However, I flirted with several other subjects along the way, including history (my undergraduate major), journalism, and geology. Before going to graduate school, I taught high school math for six blissful years in San Francisco and Colorado Springs.

I have expended much energy over the years on math circles and math contests, both locally and nationally. Recently, I helped to form a math circle in San Francisco that targets underrepresented populations and also educates teachers in parallel with their students. I have also just finished the second edition of my book *The Art and Craft of Problem Solving* (Wiley, 1999) which has been highly acclaimed by both students and teachers.

Here are four problems, all rather difficult. Don't worry about solving them—yet—but you may enjoy thinking about them.

1. The sum of the squares of three integers is equal to twice their product. What are the integers?
2. Twenty-three people, each with integral weight, decide to play football, separating into two teams of eleven people, plus a referee. To keep things fair, the teams chosen must have equal *total* weight. It turns out that no matter who is chosen to be the referee, this can always be done. Prove that the twenty-three people must all have the same weight.
3. A standard 52-card deck of cards is shuffled, and you draw cards from the top until you see an ace. On average, how many cards will you draw?
4. Consider the following experiment: First a random number p between 0 and 1 is chosen by spinning an arrow around a dial that is marked from 0 to 1. Then an unfair coin is built so that it lands ``heads up" with probability p. This coin is then flipped 2000 times, and the number of heads seen is recorded. *What is the probability that exactly 1000 heads were recorded?*