Spectral data for Higgs bundles

organized by
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Workshop Summary

Higgs bundles and their spectral data have recently started to receive attention from researchers in very different areas of mathematics and physics, and thus the workshop was planned in response to the exciting developments and interest around the notion of Higgs bundles. The purpose of the workshop was to bring the leading key players of each field to find novel perspectives from which to study Higgs bundles and its applications, and to inspire young mathematicians in the area.

The goal of the workshop was to identify new relations between previously unrelated areas of mathematics and physics. By obtaining a deeper understanding of spectral data for Higgs bundles, we hoped to be able to make substantial contributions to different research areas, among which we distinguished the Langlands program, wild Higgs bundles and Topological Quantum Field Theory.

During the workshop we had two lectures each morning, problem sessions during two afternoons (Monday and Thursday), and group discussions the remaining afternoons. Whilst some talks were closely related to one of the three main themes (which we shall describe bellow), some other talks were given with the intent of inspiring new interconnections amongst research areas and participants. In particular, David Baraglia spoke about automorphism groups of the moduli space of Higgs bundles, Andy Neitzke spoke about Higgs bundles and spectral networks, and Justin Sawon spoke about special Lagrangian fibrations.

When participants were asked about the benefits of the workshop, we were told that "*the workshop was informative, fun and helpful*". Moreover, several people have told us that they have had productive conversations and hope to collaborate with several fellow participants: in fact some participants have already got an invitation for a possible collaboration and a contact for future work.

It should be mentioned that the number of female participants (and the talks that these delivered during the weekend workshop preceding the AIM meeting) was very well received - as an example, we received a very pleasant message from a participant saying "*I appreciated the emphasis on young/female researchers. I think this type of forum has an important role to play in the development of the field.*". Finally, another participant told us "*We enjoyed a collaborative environment with experts in the field exchanging generously their knowledge which is quite an exceptional situation. ... People were also dynamic and energetic which also helped for a good work.*"

In what follows we shall describe three of the main directions pursued during the workshop, but one should note that there were several others which originated new collaborations and research directions.

Langlands program

During the second morning Laura P. Schaposnik gave the introductory talk on Higgs bundles and the Langlands program, which was followed by the one of Tony Pantev on T-duality - these talks inspired several questions during the second afternoon which were then considered during the week.

During the week we benefited from talks of other key researchers in the are, which were of invaluable inspiration: Sergei Gukov explained the appearance of branes in the moduli space of Higgs bundles and T-duality from the physics perspective, and David Nadler explained how the Nadler group which appears in a conjecture concerning (B, B, B)-branes in work of Baraglia-Schaposnik.

One particular problem studied was the construction of (B, B, B)-branes as dual of (B, A, A)-branes in the moduli space of Higgs bundles, done from a physics perspective by Lara Anderson and Justin Sawan. Simple examples were explored in detail, which should proved fruitful for further studies.

Another problem studies was the way in which branes lie inside the Hitchin fibration, and how their dual branes lie there, along the lines of the work of Baraglia-Schaposnik. This was done by David Baraglia, Steve Bradlow and Laura Schaposnik. Advances were made in some particular cases related to three manifolds, which should result in a research paper in the near future.

Wild Higgs bundles

Philip Boalch gave the introductory talk on Wild Higgs bundles during the first day, and as in the case of TQFT, during the first afternoon some problems resulted of the seminar which were then considered during the week.

One particular case which is currently being studied by Ana Peon Nieto, Refe Mazzeo, Laura P. Schaposnik, Olivia Dumitrescu and Eyal Markman is the natural relations between Higgs bundles on smooth and singular curves. Advances were made using the work of Bradlow-Schaposnik to link Higgs bundles on a decomposable surface with a smooth one. We hope to be able to write a paper with the technology developed during the week, and which can be applied to other situations.

Topological Quantum Field Theory

Jørgen Ellegaard Andersen gave the introductory talk on Topological Quantum Field Theory during the first day of the workshop, which set up the background to start working on open problems in that area. During the first afternoon several problems were suggested by the audience in relation to his talk, of which a few were selected by groups to work on during the week.

During the remainder of the week, Andersen together with Nawata, Rembado, Bradlow, Schaposnik and others looked at ways in which one could understand quantisation in the case of Higgs bundles over genus 1 curves. Several were made in this direction, and currently Rembado is writing up the notes from these days.