Singular geometry and Higgs bundles in string theory

organized by
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Workshop Summary

One of the most interesting aspects of the duality revolution in string theory is the understanding that gauge fields and matter representations can be described by the intersection of branes. Since gauge theory is at the heart of our description of physical interactions, it has opened the door to the geometric engineering of many physical systems, and in particular those arising from Higgs bundles, whose moduli spaces have become a principal source of physically-relevant branes.

In an effort to consolidate and disseminate the variety of different techniques, heuristics, and approaches that have been applied to the study of Higgs bundles and spectral data in recent years by the mathematics and physics communities, we organized the AIM Workshop *"Singular geometry and Higgs bundles in string theory"* and prepared a brief article (https://arxiv.org/pdf/1710.08453.pdfarXiv:1710.08453) where we presented a short survey on these subjects, as well as a collection of open problems and ideas revolving around them, which served as starting point for our AIM workshop. The goal of the workshop was to identify new relations between previously unrelated areas of mathematics and physics surrounding Higgs bundles and singular geometries. By obtaining a deeper understanding of spectral data for Higgs bundles, we hoped to be able to make substantial contributions to different research areas, among which we distinguished the Langlands program, Calabi-Yau geometry and String Theory.

During the workshop we had two lectures each morning, problem sessions during two afternoons (Monday and Thursday), and group discussions the remaining afternoons. Whilst some talks were closely related to one of the two main themes (which we shall describe bellow), some other talks were given with the intent of inspiring new interconnections amongst research areas and participants. In particular, during the last two days we had a varied set of speakers: Patrick Jefferson (PhD Student at Harvard) spoke about the general relations between String Theory and Higgs bundles, Ruxandra Moraru (Professor at Waterloo) spoke about Higgs bundles and generalised complex geometry, Murad Alim (Junior Professor at Hamburg) spoke about Higgs bundles and tt^*-equations, and Qiongling Li (Post-doc at Caltech/Aarhus) spoke about Higgs bundles and harmonic metrics.

In the last few weeks, we received several emails thanking us for the meeting, many saying it was the nicest workshop they've been to, and expressing how we had achieved a perfect balance between mathematics and physics (e.g., we were send short lines like this one: "*A note to say thank you for last month's AIM workshop: I found it very interesting and inspiring!*"). Moreover, several people have told us that they have had productive conversations and hope to collaborate with several fellow participants: in fact some participants

have already visited L.P. Schaposnik in November, following the new contacts established at AIM.

It should be mentioned that the number of female/minority participants (and the talks that these delivered during the week) was very well received - in particular, we had 50% minority speakers, 75% female organizers (100% minority organizers), and approx. 30% female participants. Among the speakers were two of the organizers, Laura Fredrickson (Stanford) who introduced the subject of the workshop in the fist talk, and Mboyo Esole (Northeastern), who gave an introduction to one of the particular research lines proposed, involving Calabi-Yau geometries and Higgs bundles with singular spectral curves. In what follows we shall describe some of the main directions pursued by participants during the workshop, but one should note that there were several others which originated new collaborations and research directions.

Hitchin systems and String theory.

During the second morning Sergei Gukov gave an introductory talk on Higgs bundles and branes within Sting theory, and suggested some open questions related to certain $\mathcal{N} = 3$ theory that could possibly arise through meromorphic Higgs bundles. His proposed ideas were further investigated during Monday afternoon, and his talk was complemented by Murad Alim, Florian Beck and Patrick Jefferson's talks on String theory and Higgs bundles, during the second half of the workshop, and from all of these several new paths of research were opened.

One particular line of research that was opened, and for which many advances were made, appeared through Murad Alim's talk on tt^* equations: indeed, his group believes that enough advances have been made towards a paper. The topological anti-topological fusion or tt^* equations were put forward by Cecotti and Vafa in the context of $\mathcal{N} = 2$ supersymmetric theories in two dimension and provide the data of a flat connection on the moduli space of the theory which is considered. When these theories are realized as nonlinear sigma models into a target space X, the tt^* equations endow X with the structure of a special Kähler manifold. The equations bear striking resemblance with Hitchin's equations associated to Higgs bundles and they are indeed expected to provide a generalization of the classical theory, albeit explicit examples of the correspondence spelled out in detail are lacking in the literature. Following a talk on tt^* equations, a group of workshop participants set out to seek an explicit correspondence in an example of how the data of tt^* equations can be translated into the Higgs bundle side. They succeeded in doing so by first recognizing the relations to tt^* equations in a recent paper of Fredrickson and Neitzke and using this knowledge to understand what the correspondence should be for tt^* equations attached to a family of elliptic curves which was discussed in the talk.

Higgs bundles and algebraic geometry.

Several of the introductory talks were given on topics related to singular algebraic geometry and Higgs bundles: in particular, an introduction to Higgs bundles on singular curves was delivered by Marina Logares (Plymouth), and an overview of Higgs bundles whose spectral curves are singular was given by Steven Rayan (Saskatchewan). These research talks led to many interesting problems and open questions, which the participants continued to think about in the group sessions during the afternoons of the workshop.

The group of Pablo Solis, Harold Williams, Marina Logares and Vivek Shende considered the moduli space of generalised Higgs bundles on X, which are in birational correspondence with Higgs vector bundles on Y by work of Bhosle, where Y is a nodal curve and $p : X \to Y$ its normalization. The theory of microlocal sheaves provides a candidate for the Betti moduli space of Y, in the sense of non-abelian Hodge theory, and thus the group considered an approach to establishing a nonabelian Theorem in this nodal setting by pursuing a decorated version of Simpson's parabolic nonabelian Hodge correspondence on X. Evidence for such a picture comes from certain appealing similarities in the extra data required to pass from singular objects on X to objects on Y . The group of Murad Alim, Rodrigo Barbosa, Florian Beck, Shuli Shen, and Szilárd Szabó considered the *realtionship between meromorphic Hitchin systems and Calabi-Yau integrable systems*. It is known that such a relationship holds for holomorphic Hitchin systems (i.e. with holomorphic Higgs field) and with structure group a simple adjoint or simply-connected complex Lie group. The relation between meromorphic Hitchin systems and Calabi-Yau integrable systems has been worked out by Diaconescu, Donagi and Pantev (DDP) in the case where the structure group is a simple adjoint complex Lie group of with Dynkin diagram of type A or D (this is still work in progress). Type A and D are related to type B and C by folding, and thus the group first considered these cases. As a start, Florian explained which CY3s give rise to (holomorphic) Hitchin systems with structure group $SO(5, \mathbb{C})$ (so B_2). After that Szilárd gave a description of the spectral data for meromorphic (not necessarily tame) Hitchin systems. This is important because it will be foundational to construct the corresponding CY3s.

Finally, the group of Patrick Jefferson, Ruxandra Moraru, Steven Rayan, and Alex Takeda worked on a problem motivated by a question posed by Patrick Jefferson, who is interested in finding solutions to a specific Hitchin-type system that involves two commuting Higgs fields ϕ_1 and ϕ_2 on a Riemann surface Σ, taking values in the underlying vector bundle E twisted by the square root of the canonical bundle $K_\Sigma^{1/2}$ of Σ, that are invariant under the action of a finite subgroup Γ of $SU(2)$. After one week, they have a very concrete picture of the problem and made excellent progress in several directions. They feel that they can have a complete description of the moduli spaces of stable (parabolic) vector-valued Higgs bundles on a Riemann surface, as well as of the corresponding solutions to the Hitchin- type system, within a few months (which will lead to at one paper). After that part of the problem is completed, they plan on investigating the non-abelian Hodge Theory side of the problem.