# Branching problems for unitary representations

organized by
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Workshop Summary

The *branching problems* ask for how irreducible unitary representations of $G$ decompose when restricted to a subgroup $H$. The decomposition of tensor products is an example of branching problems. Fourier analysis arises naturally in branching problems from $SL(2,\mathbb{R})$ to abelian subgroups $\mathbb{R}$ and $SO(2)$. Littlewood-Richardson rules are combinatorial algorithms for finite dimensional branching laws. The so-called Theta correspondence in number theory is also closely connected to branching problems. The lectures in the morning reflected the wide range of techniques and problems in this field. They were very well prepared and served in an excellent way the purpose of providing common background and motivating problems for the participants.

T. Kobayashi gave two survey lectures on the general topic of branching problems, with emphasis on decomposing unitary representations of real reductive Lie groups. He explained the state or the art on analysis on homogeneous spaces (as a dual notion to the restriction), gave new examples on various bad phenomena that arise when restricting representations to subgroups, and suggested a good framework in which one could expect branching laws to be made explicit. For representations attached to hyperbolic orbits the application and the limitation of Mackey theory were explained. From algebraic viewpoints, he explained his theory of admissible restriction (discretely decomposable with finite multiplicities) for $(\mathfrak{g}, K)$-modules, and specific results were given, in particular the new classification of admissible restriction of cohomologically induced representations which are attached to elliptic orbits in the orbit philosophy. For minimal representations attached to nilpotent orbits, an example of branching laws was illustrated from two view points, the one is geometric (restriction from conformal group to isometry subgroup) and the other is algebraic (sea-saw rule in the theta correspondence).

M. Duflo lectured on functorial aspects of the orbit philosophy between the set of irreducible unitary representations (the unitary dual) and the set of coadjoint orbits. In particular he explained the relation to branching rules, giving the explicit details in the case of simply connected nilpotent Lie groups. This case is the most complete and has been a major motivation for subsequent work. He also covered almost algebraic groups and recent results on branching laws for discrete series representations of real reductive groups.

J. Vargas continued with explicit examples of branching laws for discrete series representations of real reductive groups; he explained the joint work with M. Duflo and their "condition C" which gives a sufficient condition for admissible restrictions to reductive subgroups as well as combinatorial formulas for multiplicities which are derived from the Heckman-Duflo-Vergne formula for coadjoints orbits as symplectic manifolds.

R. Howe surveyed his theory of dual pairs, starting from the representation theory of the Heisenberg group. He constructed the Schrödinger model of the metaplectic representation (or the oscillator representation), and extended it to a holomorphic semigroup. He then explained the Theta correspondence for dual reductive pairs, which is an important tool both in constructing representations and in understanding branching laws. In the so-called stable range this gives an injection of the unitary dual of $O(p, q)$ into that of the metaplectic group.

A. Paul lectured on results about dual pairs and their relevance for studying the unitary dual, in particular the problem of finding the exact correspondence between the admissible duals for a dual pair in terms of the Langlands parameter, and the question of preservation of unitarity in the correspondence. Many examples were given, including cohomologically induced modules.

Y. Oshima explained his recent joint with T. Kobayashi concerning admissibility of representations restricted to a symmetric subgroup. He explained some important criteria, some necessary and some sufficient, for admissible restriction by T.Kobayashi, leading to a complete classification of the triples $(G, H, )$ for which the corresponding Zuckerman module $A$ is discretely decomposable with respect to $H$. Also some explicit examples were given.

P.-E. Paradan lectured on the theme, "Quantization commutes with reduction" giving the definition of geometric quantization via Dirac operators as an equivariant index. The main theorem summarizes many years of progress in this area, in particular the work of Paradan using K-theory. Branching laws of discrete series representations to maximal compact subgroups were explained in this framework. Finally some steps in the proof were explained such as localization and suitable Morse functions.

T.N. Venkataramana presented some applications of branching laws to number theory, concentrating on cohomology of locally symmetric spaces and the restriction of cohomology classes to locally symmetric submanifolds (modular symbols). As a result, by finding branching laws for Zuckerman's module $A_{\mathfrak{q}}$ (representations with non-zero $(\mathfrak{g}, K)$-cohomology), one obtains results about non-vanishing restrictions of cohomology classes.

M. Pevzner lectured on analytic aspects of the branching of minimal representations of the general linear group to symmetric subgroups, in particular the symplectic group. He explained the role of generalized Hardy spaces, the Weyl calculus, the Jacobi group and its representation theory, and the metaplectic representation. As a result he gave new irreducibility results for a unitary generalized principal series for the symplectic group induced from a Heisenberg parabolic subgroups.

The work in smaller groups turned out to be quite interesting and productive; specific problems were addressed and real progress was made.