Control and optimization of open quantum systems for information processing

organized by
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Workshop Summary

This workshop was held at the American Institute of Mathematics (AIM) in Palo Alto, CA from June 21-25, 2010. The workshop enabled researchers from three distinct quantum-information-science research communities to interact and exchange ideas. Specifically, these fields were dynamical decoupling (Lie-algebra based), optimal control (topology/variational-calculus based) and quantum error-correction (algebraic-coding based). Participants included professors, post-doctoral fellows, and graduate students from academia as well as government and private-institution researchers. By many measures, the workshop was successful; the tutorials were interactive and enlightening and the collaborative discussion sessions were lively and engaging. Verbal feedback from the participants was universally positive and several participants reported that they formed new cross-cutting scientific relationships as a result of the workshop. In this report, we summarize the workshop's outcomes, including a list of future research directions indicated by participants.

1. Workshop Motivation

The field of quantum information involves the complex task of designing and effectively manipulating multi-qubit systems. However, this problem is beset by significant difficulties, such as corruption of quantum information caused by decoherence and under-utilization of classical controls, which are a rich resource for the manipulation and protection of quantum systems. Finding solutions to the problem of decoherence, resulting from the unavoidable interaction of a quantum system with its environment, is one of the most critical challenges impeding practical realizations of a quantum information processor (QIP). Current strategies for decoherence management were developed by researchers from three distinct communities within the quantum information sciences (QIS), namely, dynamical decoupling (DD), optimal control (OC), and quantum error correction (QEC). This workshop enabled researchers from these communities to interact and generate novel solutions. By combining the expertise of each community, the workshop explored the decoherence problem from a multi-disciplinary perspective.

DD, which has its foundations in the spin-echo effect pioneered by E. Hahn [14] and average Hamiltonian theory [30, 31] for nuclear magnetic resonance spectroscopy, seeks to decouple a system (e.g., a QIP) from its environment, thereby extending the lifetime of quantum memory channels. This is accomplished by applying a sequence of control field pulses to the system, essentially producing an approximated time reversal of the system-environment interaction. Designs of DD pulse sequences often employ only the most basic information on

$$\min_C \|U(C, t_\mathrm{f}) - V_\mathrm{sys}^\mathrm{target} \otimes V_\mathrm{env}\|$$

subject to

$$\dot{U}(C, t) = -\mathrm{i}H(C, t)U(C, t)$$
$$H(C, t) = H_\mathrm{sys}(C, t) + H_\mathrm{env} + H_\mathrm{int}$$

FIGURE 1. The main topic of the workshop, expressed mathematically. In words, this says that the object is to minimize the distance between the actual and desired unitary quantum transformations subject to Schrödinger's equation driven by a controllable Hamiltonian that acts on a system and its surrounding environment.

the system-environment interaction, without detailed knowledge of the system and environment Hamiltonians. Improvements can be added to DD by concatenating the pulse sequences [17], thereby increasing their complexity and duration. Typically, DD pulse sequences are designed *a priori*, which corresponds to the so-called "open-loop" control process; however, a "closed-loop" laboratory optimization of pulse intervals in a DD sequence was recently demonstrated [3].

OC is a general procedure employing iterative approaches such as adaptive feedback and variational calculus to design controls for any physical objective that can be formulated mathematically [7, 28], e.g., preservation of quantum memory channels or construction of high-fidelity quantum operations. It is also particularly well-suited for optimization of multiple objectives, while including constraints on admissible controls. Applying OC methods to control of quantum phenomena has developed during the last two decades into an important field of interdisciplinary research, with a rich history of theoretical developments and successful experiments [23, 2, 26, 6]. OC can be employed either in the open-loop fashion (with a model-based theoretical control design directly applied in an experiment), or in a closed-loop setup (with controls adaptively optimized in the laboratory, guided by measurement outcomes which are fed back to a learning algorithm). Recent studies strongly suggest that development and application of concepts, methods, and algorithms of OC to QIS is crucial for the progress of this field. In fact, applying OC to manage dynamics of open quantum systems and protect quantum information from decoherence has become increasingly popular in the last few years (e.g., see Refs. [35, 5, 12, 16, 32, 24, 13]). Results obtained in these works illustrate how practical quantum computing can be greatly facilitated by OC and reveal interesting physical insights through the discovery of effective control mechanisms.

Assuming that errors occur independently on individual physical qubits and/or only weak (spatial) correlations exist in multi-qubit errors, QEC employs redundant encoding of quantum information, producing logical qubits that can be restored from errors [8, 27, 18, 11]. The errors are identified by measurements on additional (ancilla) qubits, so that the information in the logical qubit is not destroyed. Once identified, the errors can be corrected. Since operations performed on the qubits are conditioned upon measurement outcomes, QEC can be considered to be a real-time closed-loop process. Like in the case of DD, codes for QEC also can be improved by concatenation, which, however, polynomially increases the resource cost in terms of the number of physical qubits and operations required to perform a fault-tolerant quantum computation (FTQC). In addition, convex optimization methods [4] were also proposed for improving the QEC performance [25, 20, 19].

As a moniker for integrative control approaches that combine the desirable properties of the methods described above, we suggest the term "hybrid quantum control" (analogous to "hybrid quantum devices" in Ref. [29]). Figure 1 expresses the primary focus of the workshop mathematically.

2. Daily tutorials

Because none of the participants are experts in all three research areas, the schedule of the tutorials for the first two days was selected to systematically educate and inform the participants and guide the initial afternoon collaborative discussions. Tutorials for the rest of the week were selected based on the needs of the workshop.

(1) Monday
   - (a) Herschel Rabitz – *"Controlling Quantum Dynamics Phenomena with Shaped Laser Pulses Acting as Photonic Reagents"*
   - (b) Rebing Wu – *"Quantum Optimal Control Landscapes: A 'Simplicity' Theory"*

(2) Tuesday
   - (a) Frank Gaitan – *"Quantum Error Correcting Codes"*
   - (b) John Preskill – *"Fault-Tolerant Quantum Computation"*

(3) Wednesday
   - (a) Daniel Lidar – *"High fidelity quantum computation via dynamical decoupling"*
   - (b) Kaveh Khodjasteh – *"Dynamical Quantum Error Correction: Hamiltonian Open-Loop Quantum Control of Open Quantum Systems"*

(4) Thursday
   - (a) Daniel Lidar – *"Decouple then Compute"*
   - (b) Götz Uhrig – *"Universal Dynamical Decoupling"*
   - (c) Todd Brun & Andrew Landahl – *"Continuous-Time Quantum Error Correction"*

(5) Friday
   - (a) Robert Kosut – *"System Identification for Control and Error Correction of Quantum Information Processing Systems"*
   - (b) Alireza Shabani – *"Compressed Quantum Process Tomography"*

3. Collaborative discussion sessions

(1) Monday

(a) Optimal control and dynamical decoupling
(b) Optimal control and quantum error correction

Note: both sessions were lead by OC experts.

(2) Tuesday

(a) Quantum error correction and dynamical decoupling
(b) Quantum error correction and optimal control
Note: continuing the discussions from Monday, both sessions were lead by QEC experts.

(3) Wednesday

(a) Fault-tolerant adiabatic quantum computation
(b) Experimental constraints and limitations of model assumptions
(c) Robust formulation of quantum optimal control I

(4) Thursday

(a) Continuous-time quantum error correction & real-time feedback control
(b) Robust formulation of quantum optimal control II

(5) Friday

(a) Prospects for combining dynamical decoupling, optimal control, & quantum error correction
(b) Group summary of workshop activities & future directions

4. Open issues for future research

4.1 Integrating dynamical decoupling, optimal control, and quantum error correction

Successfully combining DD, OC, and QEC for improved control of open quantum systems represents a tremendous accomplishment for this workshop and QIS in general. We identified several possibilities for integration of these approaches. Perhaps the most straightforward possibility involves a combination of pulse-shaping by OC with the DD+QEC method recently proposed by H. K. Ng *et al.* [22]. In this context, OC could be used to design all physical controls that are required for the DD sequences and QEC operations. As a result of the workshop, OC, in general, was identified as a hardware, rather than software, component, based on the many theoretical and experimental control studies involving system-specific models. To clarify this point, consider that DD/DCG and QEC were developed from a set of general assumptions and their implementation does not require much detailed system, control, or environment information, whereas OC (numerical in practice) requires the specification of system and environment parameters to solve the relevant dynamical equations to generate control fields and fidelities for a given objective. Combining the pulse-shaping capabilities invoked by OC with the analytical formalism of DD/DCGs is another promising hybrid method. In addition to utilizing the underlying quantum control landscape structure [15] and the effective optimization algorithms for locating control fields, control theory for quantum-mechanical objectives may be reformulated to design controls that are robust to control/system uncertainties.

4.2 Continuous-time quantum error correction and real-time feedback control

After all three control strategies (DD, OC, and QEC) were introduced and discussed, participants expressed interest in continuous-time quantum error correction (CT-QEC) and real-time feedback control (RTFC). Most of Thursday was devoted to exploring these topics. After reviewing some of the contributions to CT-QEC and RTFC in the published literature [10, 1, 9, 21], we identified several open questions and some potential solutions. One of the most significant issues involves the "back-action" effect from measuring the state of a quantum system. Unlike RTFC for classical systems, modeling the effect of measuring the state of a quantum system involves a stochastic formulation of the ensuing quantum dynamics. Unfortunately, the computational time required to solve these dynamics in real time for RTFC is incompatible with the time scales of most quantum systems. Unless simplifications are made in the numerical simulations of the dynamics or limiting approximations/simplifications are incorporated, implementing CT-QEC and RTFC will be limited by these incommensurate time scales. Additional open questions CT-QEC and/or RTFC include the following.

(1) What is the optimization objective?
(2) Given the outcome/signal of a (weak) quantum measurement, what is the update rule for the control?
(3) How can CT-QEC and/or RTFC be combined with DD and other logical operations?
(4) How does one perform a threshold analysis for CT-QEC? Such an analysis is necessary to incorporate CT-QEC/RTFC into the standard FTQC framework.

4.3 Robust control strategies and multi-objective optimization

During the workshop, there also was interest in mathematical formulation of robust control strategies for QIS objectives, including methods involving multi-objective optimization. We discussed linear optimal control as the modern "workhorse" of classical control theorists and limitations on applicability of this formalism to quantum-mechanical dynamics/objectives. Specifically, we discussed the "small gain theorem" [33, 34] and limitations of linearized feedback control theory for a nonlinear system. It was during the "OC for QEC" collaborative session on Monday that CT-QEC and RTFC were initially discussed as a natural union of OC and QEC. We also considered types of objective functionals applicable for control of quantum information systems, with the goal of simultaneously maximizing both fidelity and robustness to control noise. Because maxima or minima of the objective functional determine control optimality, the development of an appropriate objective functional is important for control and optimization (for both classical and quantum systems) [28]. After two collaborative sessions on robust control theory, we formulated the optimal robust control problem as a multi-objective control, considering noise and/or uncertainty from control fields and system parameters. This formulation needs to be further explored analytically and numerically to determine the topology of the control landscape and its effect on robustness for some example problems. Future research in this direction would involve expressing the fidelity error in terms of the control field characteristics and noise spectrum. Such an expression would make it possible to optimize robustness for a given random noise process with particular spectral characteristics.

4.4 Integrating optimal control theory and fault-tolerant quantum computation

The workshop also helped to reveal the need to combine FTQC and OC. Previous work has shown that DD and QEC can enable FTQC, but the corresponding analysis for OC remains a largely open and unexplored problem. Pending the outcome of this analysis, it would be interesting to explore FTQC in the context of a hybrid "DD+OC+QEC" quantum control scheme, similar to what was done by H. K. Ng *et al.* [22]. Development of error-threshold-based objective functionals for optimal control was suggested.

4.5 Additional objectives and open issues

(1) Quantum gate fidelity balance between optimal and robust control.
(2) Benefits of random DD sequences and DD acting on the environment.
(3) Realistic quantum device engineering guided by theory.
(4) Validation for quantum devices (determining/improving error measurements and noise models), including a set of "community problems".
(5) Relevance of worst-case error versus average/probable fault-tolerance.

5. Concluding remarks

In the general field of quantum control, including the relatively new area of hybrid quantum control, much work remains! Because of the (a) demanding levels of operational fidelity required for fault-tolerant quantum computing, (b) uncertainty in system parameters, (c) noisy classical controls, and (d) unavoidable system-environment coupling, it is *crucial* to continue developing improved hybrid methods for integrating DD, OC, and QEC to successfully realizae a scalable QIP. This workshop provided a forum to establish some of the foundations of hybrid quantum control and cultivate new collaboration opportunities.

With interactive multi-disciplinary tutorial presentations in the mornings and dynamic collaborative sessions in the afternoons, this workshop provided an important service to the QIS research community. The largely extemporaneous/spontaneous structure AIM workshops is quite unique; the agenda evolved daily according the interests of the participants.

Throughout the week, many new relationships for potential future collaborations were formed as a result of illuminating discussions between researchers from the different communities DD, OC, and QEC.

We express our deepest gratitude to the AIM staff, who were wonderful hosts. Their guidance and suggestions in helping us organize and run a successful workshop is greatly appreciated. We also thank all of the participants for their active involvement, especially those who presented tutorials and lead collaborative discussion sessions.