Nonhomogeneous boundary-value problems for nonlinear waves

organized by
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Workshop Summary

Activities

The participants of the workshop consisted of experts in partial differential equations, numerical analyst and oceanography. The workshop began on Monday morning with talks by Prof. Jerry Bona, giving a general picture of some of the current research in water wave theory and practice. This was followed by a lecture from Beatrice Pelloni on general theory for the kind of initial-boundary-value problems that were the focus of attention. The mornings were taken up throughout with a pair of lectures by specialists in theory, modeling and applications, control theory and numerical analylsis. The range of the presentations was delightfully broad, considering the narrowness of the topic.

The afternoons were given over the small group discussions whose topics varied slowly during the week. These included groups working on exact solutions, control theory, particular problems arising in water wave theory, integrability, inverse problems and the like. Some of these sessions were quite successful and led to the formulation of a set of specific problems and open issues, some of which have been the focus of further research among the participants and some of which have been settled by groups of participants.

Focus Problems

- Formulating the exact boundary conditions that should be used for models defined on a quarter plane
- Investigate the integrability of various Boussinesq system
- boundary-value problems for the heat equation
- Decay of solutions of KdV-Burgers equation on the half line
- Strichartz estimates in the presence of inhomogeneous Neumann and Direchlet data
- Transparent boundary conditions for various wave equations
- The inverse wave maker problem
- Controllability of the model problem

$$u_t + u_x + uu_x + u_{xxx} = 0,\ 0 < x < L$$
$$u(x,0) = 0 \quad \text{and} \quad u(0,t) = h(t)$$

- Wellposedness of

$$u_t + u_x + uu_x + u_{xxx} = 0, \quad 0 < x < L,$$

with auxiliary data

$$u(x,0) = \phi(x),\ u(0,t) = 0,\ u(L,t) = 0,\ u_x(0,t) = u_x(L,t)$$

- Global wellposedness of

$$u_t + u_x + uu_x + u_{xxx} = 0$$

  under the conditions

$$u(x,0) = \phi(x),\ u(0,t) = 0,\ u(L,t) = 0,\ u_{xx}(L,t) = 0$$

- Search for integrable versions of the *abcd*–systems and associated Painleve structures, explicit 2D solutions and stability
- Asymptotic periodicity — what gives?
- Prove or disprove blow up of

$$u_t + u^k u_x + u_{xxx} - u_{xx} = 0 \quad u(x,0) = \phi(x)$$

  for $k \geq 4$ (Conjecture: the result will be there same as without the dissipative u_{xx} term)
- Stricthart estimates for wave equation and Schrodinger equations on the half plane and in domains with concave boundaries
- Appropriate boundary conditions for 2-space dimension Boussinesq systems
- derivative NLS on the half line

Post-workshop activity

While there were existing collaborations among some of the participants, several new ones were engendered by the workshop. And some of the problems that arose in discussions during the workshop have become active areas of research by subgroups of the participants. Here are some particularly interesting examples.

Shu-Ming Sun is leading a group that is working on the problem of how to formulate the exact boundary conditions with the Euler equations in a quarter plane.

On the last day of the workshop, Lionel Rosier, Shu-Ming Sun and Bingyu Zhang started an extended project on decay of solutions of the KdV-Burgers equation on the half line. The goal is to obtain the same (or similar) results Amick, Bona and Schonbek established many years ago for the KdV-Burgers equation on the whole line. Lately, this project has begun to look promising.

Bernard Deconinck, Min Chen and one of Deconinck's Ph.D. students are continuing the effort to apply Painleve analysis to *abcd*–systems to investigate their integrability. This project got started during the AIM meeting .

Deconinck, Chen and Shiels have just finished a paper on the heat conduction problem they were working on during the workshop. It has been submitted for publication.

A related project is that of Shu-Ming Sun and Bernard Deconinck searching for the exact solutions of *abcd*–systems.

Barbara Karakiewicz, Bona and their collaborators are finishing up a script on sediment transport whose perspective was aided by participation in the workshop.

Bernard and his students are continuing working on the heat equation issue that arose in the meeting.

Jonatan Lenells and Jerry Bona have apparently solved the KdV–quarter plane problem using the inverse scattering transform method of Fokas and Pelloni. A draft manuscript has been produced.

Beatrice Pelloni has organized a special session and the upcoming meeting in Madrid around the ideas coming out of the workshop.

Concluding Remarks

Judging by the subsequent activity, the workshop was a success. The intensity of interactions during the workshop supports this conclusion.

The organizers want to tender their heartfelt appreciation of AIM and the AIM support staff for providing the basic structure around which the workshop ran so smoothly. We would do it again in a second!