Mixing and nonlinear stability

organized by
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Workshop Summary

1. Overview

The goal of the workshop was to bring experts and younger participants together to expand the mathematical analysis of mixing phenomena arising in fluid mechanics and kinetic theory. The specific focus was on the relationship between mixing and nonlinear stability problems, that is, how the fluid *mixing itself* changes the dynamics. In fluid mechanics, the mixing effect is sometimes called *inviscid damping*, together with the closely related *mixing-enhanced dissipation* in the viscous case. These mechanisms are connected to the metastability of coherent structures at high Reynolds number, such as shear flows and vortices (see e.g. [MelanderEtAl87,SchecterEtAl00,GilbertBassom98,BM95,CerfonEtAl13,BM13,BMV14] and the references therein). This is expected to be important for understanding the stability of hurricanes and other weather phenomena [MontgomeryKallenback1997,SmithMontgomery1995] as well as potentially playing a general role in organizing the flow in 2D turbulence and large-scale atmosphere/ocean dynamics [Gilbert88,Gilbert1993,GilbertBassom98,Shnirelman12,GSV13]. In plasma physics and galaxy dynamics, the related mixing effect known as *Landau damping* has long been recognized as a fundamental stability mechanism in nearly collisionless kinetic models (see e.g. [Binney-Tremaine,Ryutov99,villani2010,MouhotVillani11,BMM13] and the references therein). Despite its fundamental physical relevance and importance in practical settings, the mathematical analysis of these mixing phenomena in nonlinear PDE is still almost non-existent due to subtle regularity issues connected with unusual nonlinear resonances and even often a lack of clear understanding of linear mixing phenomena.

The workshop succeeded in multiple senses. First, the workshop stimulated a number of very interesting discussions between participants, including a number of new, surprising directions. Second, the workshop provided an opportunity for participants, especially those new to the field but also the experts, to learn a great deal of new techniques. The informal, 1.5 hour talks in the morning were especially conducive to this. Moreover, some group discussions materialized in the afternoons to provide tutorials or more in-depth discussion on certain topics, such as nonlinear resonances and KAM theory. Third, open problems were categorized and discussed as a group, providing those looking for interesting problems some context and ideas. The organizers received positive feedback from both senior and junior participants. We felt that overall the workshop was a solid success.

2. Lectures and group discussions

- Clement Mouhot introduced Landau damping and gave an overview of existing literature in kinetic theory.

- Zhiwu Lin discussed mixing-enhanced dissipation and recent results of his collaborators and him on applications to nonlinear meta-stability results in 2D fluid mechanics.
- Benoit Grebert discussed the use of KAM theory for infinite dimensional Hamiltonian PDEs. As an example, he discussed the work of his collaborators and him on the construction of quasi-periodic solutions to the semilinear beam equation. The interest of this topic was to stimulate discussions about whether or not such quasi-periodic solutions could be constructed in related settings in fluid mechanics and kinetic theory.
- Zaher Hani discussed results of his collaborators and him on norm growth in the nonlinear Schrodinger equations. The connection with underlying nonlinear resonances was made clear.
- Thierry Gallay discussed work in progress regarding spectral asymptotics in singular limits and mixing-enhanced dissipation near 2D vortices.
- Michele Coti Zelati discussed recent work of his collaborators and himself on mixing-enhanced dissipation in linear advection-diffusion equations with stochastic forcing.
- David Gerard-Varet discussed recent work of his collaborators and himself regarding stability and Landau damping in the Kuramoto equations.
- Natasa Pavolvic discussed recent work of her and her collaborators regarding quantum many-body systems, related to several of the discussion groups.
- Gene Wayne discussed recent work of himself and his collaborators on Taylor dispersion in shear flows.

There were a number of group-wide discussions as well as smaller, more focused work groups. Here are some of the topics that were discussed in these settings:

- KAM theory and quasi-periodic solutions in nonlinear Schrodinger and other dispersive equations with continuous spectra.
- Nonlinear echo resonances in Vlasov-Poisson and the 2D Euler equations, as well as some of the existing technical tools for understanding them.
- Landau damping in Vlasov-Maxwell.
- Connections to many-particle dynamics, specifically, getting a better understanding of the limit of large numbers of particles and the mean-field limit.
- Connections to dispersive equations and questions regarding the stability of solitons and scattering.
- Mixing-enhanced dissipation and inviscid damping in compressible fluids.
- Non-monotone shear flows.

3. Outcomes

Several young, as well as senior, participants expressed interests in specific problems, and it is likely several will be attempted. Moreover, several collaboration teams are attempting to continue their work, for example, the group that discussed the Vlasov-Maxwell equations. We believe that this conference will have a strong impact on the development of these ideas in fluid mechanics and kinetic theory, both through providing a good list of open problems and through providing participants with a chance to discuss and learn the topic in an informal, collaborative atmosphere.

Bibliography

[BM95] N.J. Balmforth and P.J. Morrison. Normal modes and continuous spectra. *Annals of the New York Academy of Sciences*, 773(1):80–94, 1995.

[GilbertBassom98] A.P. Bassom and A.D. Gilbert. The spiral wind-up of vorticity in an inviscid planar vortex. *J. Fluid Mech.*, 371:109–140, 1998.

[BM13] J. Bedrossian and N. Masmoudi. Inviscid damping and the asymptotic stability of planar shear flows in the 2D Euler equations. *arXiv:1306.5028*, 2013.

[BMM13] J. Bedrossian, N. Masmoudi, and C. Mouhot. Landau damping: paraproducts and Gevrey regularity. *arXiv:1311.2870*, 2013.

[BMV14] J. Bedrossian, N. Masmoudi, and V. Vicol. Enhanced dissipation and inviscid damping in the inviscid limit of the Navier-Stokes equations near the 2D Couette flow. *arXiv:1408.4754*, 2014.

[Binney-Tremaine] J. Binney and S. Tremaine. *Galactic Dynamics*. Princeton University Press (2d edition), 2008.

[CerfonEtAl13] A.J. Cerfon, J.P. Freidberg, F.I. Parra, and T.A. Antaya. Analytic fluid theory of beam spiraling in high-intensity cyclotrons. *Phys. Rev. ST Accel. Beams*, 16(024202), 2013.

[Gilbert88] A.D. Gilbert. Spiral structures and spectra in two-dimensional turbulence. *J. Fluid Mech.*, 193:475–497, 1988.

[Gilbert1993] A.D. Gilbert. A cascade interpretation of lundgren's stretched spiral vortex model for turbulent fine structure. *Physics of Fluids A: Fluid Dynamics*, 5:2831, 1993.

[GSV13] Nathan Glatt-Holtz, Vladimir Sverak, and Vlad Vicol. On inviscid limits for the stochastic navier-stokes equations and related models. *arXiv preprint arXiv:1302.0542*, 2013.

[MelanderEtAl87] MV Melander, JC McWilliams, and NJ Zabusky. Axisymmetrization and vorticity-gradient intensification of an isolated two-dimensional vortex through filamentation. *Journal of Fluid Mechanics*, 178:137–159, 1987.

[MontgomeryKallenback1997] Michael T Montgomery and Randall J Kallenbach. A theory for vortex rossby-waves and its application to spiral bands and intensity changes in hurricanes. *Quarterly Journal of the Royal Meteorological Society*, 123(538):435–465, 1997.

[MouhotVillani11] C. Mouhot and C. Villani. On Landau damping. *Acta Math.*, 207:29–201, 2011.

[Ryutov99] D.D. Ryutov. Landau damping: half a century with the great discovery. *Plasma physics and controlled fusion*, 41(3A):A1, 1999.

[SchecterEtAl00] D.A. Schecter, D. Dubin, A.C. Cass, C.F. Driscoll, and I.M. Lansky et. al. Inviscid damping of asymmetries on a two-dimensional vortex. *Phys. Fl.*, 12, 2000.

[Shnirelman12] Alexander Shnirelman. On the long time behavior of fluid flows. *preprint*, 2012.

[SmithMontgomery1995] Gerald B Smith and Michael T Montgomery. Vortex axisymmetrization: Dependence on azimuthal wave-number or asymmetric radial structure changes. *Quarterly Journal of the Royal Meteorological Society*, 121(527):1615–1650, 1995.

[villani2010] Cédric Villani. Landau damping. *Notes de cours, CEMRACS*, 2010.