Moduli Spaces of Properly Embedded Minimal Surfaces

organized by
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Workshop Summary

The conference on Moduli Spaces of Minimal Surfaces at AIM (June 6-10, 2005) had at its nucleus the problem of describing properly embedded minimal surfaces in flat three-manifolds. The organizers (David Hoffman, Matthias Weber, Michael Wolf) primary objective was based on our view of the field of minimal surfaces as it stood in mid-2005.

In the last 5 years, there have been dramatic advances in the subject. Colding and Minicozzi, in a series of perhaps 7 milestone papers (four recently published in the Annals, another submitted to the Annals), introduced strong new estimates and structural results that would change how the basic objects of study were to be regarded. These results, when combined with the rescaling techniques of Meeks and his collaborators (Rosenberg, Perez, Ros and others), focused the attention of the field away from individual surfaces to families of surfaces and new limiting objects: minimal laminations. Other singular limits were considered by Pacard and his collaborators and by Traizet, in different settings, and each had developed new and different desingularization techniques. Weber, Hoffman and Wolf had obtained results of a different character by combining deeper techniques from Riemann-surface theory with contemporary viewpoints in geometric structures.

While results that were beyond anyones expectation five years ago were achieved, the natural expectation of a synthesis—a new paradigm—has not occurred. The places where these varied techniques were assimilated and used in concert by collaborators were few, even though the results were often path-breaking. The reason for this we think is that users of one of these techniques found the 'price of entry' to the other techniques too high. There really was no single paper or line of research that combined the techniques represented in the four directions mentioned above, and even the best young people seemed to be picking a single direction to pursue.

All in all, the field was both at a moment of unique promise and a moment of unique danger; rapid advances made the solution of many longstanding conjectures possible, but it appeared to us that the subject might advance in distinct and non-overlapping directions.

Based on this analysis, a principal goal of the conference was to facilitate the entry of established experts into the new areas developed in the recent past. Much of the conference was built around this objective, with the originators of the approaches giving long introductory talks to their work. The experts were instructed to focus on techniques rather than results and to organize their talks around very basic and easily understood goals, rather than the most general and recent result obtainable.

In terms of specifics, the following were highlights:

(1) Joaquin Perez gave a 2.5 hour report on rescaling techniques and how they are used, followed by a 3.5 hour demonstration (over two days) of such use by Harold Rosenberg and a two-hour discussion of those techniques by Bill Meeks. A number of open problems were suggested.
(2) Antonio Ros gave a long overview of moduli space techniques in minimal surface theory, and Joaquin Perez and Mike Wolf gave technique talks.
(3) Martin Traizet and Frank Pacard both explained how to use their techniques to create a very simple surface, Riemann's minimal surface, described 150 years ago, but perfect as a tool for exploring their methods.
(4) Bill Minicozzi gave a day-long exposition of his work with Toby Colding on the structure of embedded minimal disks.
(5) Additional talks, suggested by the participants, were given by Rob Kusner on the structure of ends of embedded minimal surfaces, by Francisco Martin on minimal immersions into bounded domains, and by Laurent Hauswirth on adding Costa surfaces to Riemann's minimal surface.
(6) An active problem session at the end yielded a list of 14 broad problems with references that the younger participants found especially useful.

The organizers plan to make the results of the conference broadly usable by posting summaries of the sessions and links to the most recent relevant literature on a web site.

The plan worked well: at various levels, the participants reported that they felt newly comfortable with the new techniques, and many planned to attempt another reading of the core material. Numerous research problems were discussed in smaller groups. We give one example.

There is a now-classical result due to Shiffman, which says that a minimal annulus bounded by convex curves in parallel planes is fibred by convex curves (in parallel planes). It is believed that the question, Do there exist convex curves in parallel planes that bound an embedded minimal surface of higher genus?, has a negative answer. Various researchers have attacked this question with techniques described in 1)-3). In a series of conversations, it was discovered that several of these techniques could be combined in a fruitful manner. It also became known that a simple case of this question had been resolved (when the convex curves are identical vertical translates of one another) in the negative, but almost no one (including the two organizers who are interested in it) was aware of this fact(!). In this and a number of other areas, significant progress was made in understanding, and one can be confident that there will be results forthcoming whose roots trace back to this week at AIM.

Naturally, as at any mathematical meeting, the participants in this AIM workshop wished to learn the very newest results and to become acquainted with the cutting-edge problems in the field, and they also wished to renew old relationships and possibly begin or continue collaborations. We believe they got this and much more during the week.