

THEORY AND ALGORITHMS OF LINEAR MATRIX INEQUALITIES

organized by

John Helton, Pablo A. Parrilo, and Mihai Putinar

Workshop Summary

We gathered four groups of mathematicians, working on quite distinct areas: *engineering systems and control, operator theory, real algebra and optimization*. These represented well the strongest senior people working in aspects of these areas and some very promising younger people.

The dialogue was continuous, very dynamic, rich and constructive. Nobody attending knew all areas well and indeed most people there had never met.

On Monday we started the meeting with talks by the three organizers on how this group and the four areas fit together. We had a meeting where people gave 3-10 minute introductions to themselves and to mathematical questions they liked. This was day one and it did well not only on the informational side but at creating an informal atmosphere and making it clear to everyone that their talks required serious expository components. It put everyone in the position of knowing rather precisely what audience to prepare their talk for.

On Tuesday we started with a general survey of moment methods especially in relation to computational issues by Jean Lasserre. Then we had a moderated session to obtain suggestions from participants for the week's program. This flexible scheme at AIM helped us better adapt to the needs of this community and to focus on spontaneous ideas and major gaps particular to this rather unusual audience.

A breaking development, due to Konrad Schmüdgen, in non-commutative real algebraic geometry is a Positivstellensatz for partial differential operators with polynomial coefficients. Its gist is to write a positive differential operator as a weighted sum of squares of differential operators. Konrad Schmüdgen described his new theory to the full group in a talk Wednesday morning. He did not know of classical partial differential analogs of his result. By luck several operator theorists (Ball, Helton and McCullough) had worked on a strong classical analog. We had a lively session laying out these correspondences and speculating on extensions.

On Thursday we had a talk by Vicki Powers to the full group giving a very fine survey of classical real algebraic geometry.

On Friday, Dan Voiculescu gave a talk tailored to our matrix inequality conference, saying how free probability, which he invented, might give some idea of how to compute the probability that randomly generated large matrices might solve a given set of noncommutative polynomial inequalities.

Most of the week was spent with sessions, always two in parallel. A brief description of the schedule follows:

Monday. August 1, 2005

9:00 am - Overview
 2:00 pm - 3 min. talks

Tuesday. August 2, 2005

9:00 am - Jean Lasserre: Generalized problem of moments
 10:30 am - Discussion
 2:00 pm - Section I. Systems and Control (introduction)
 From systems to LMI's
 Section II. Hyperbolic Polynomials

Wednesday. August 3, 2005

9:00 am - Konrad Schmüdgen: Commutative and non-commutative Positivstellensätze
 10:30 am - Section I. Non-commutative real algebraic geometric applications
 Section II: Determinantal representations
 2:00 pm - Section I: Differential operators and Lyapunov inequalities
 Section II: Lifting LMI representations, moment methods, etc.

Thursday. August 4, 2005

9:00 am - Vicky Powers: Real algebraic geometry
 10:30 am, Section I: Fisher duality
 Section II: Model reduction
 2:00 pm - Section I: Minimum rank problems

Friday. August 5, 2005

9:00 am - Section I, Numerical methods, software, etc.
 Section II, Many variable systems interpolation, single square factorization
 10:50 am - Dan Voiculescu: Operator algebras/ free probability aspects of operator inequalities
 2:30 pm - Real algebraic geometry vs operator theory, and how to teach them
 3:30 pm Open problems

We have isolated a good number of open problems. They will guide the research in the new born area of non-commutative real algebraic geometry. On Friday afternoon we discussed these as well as pedagogical issues, in particular how to help the future graduate students access this field. A couple of advanced students had the opportunity to present their work and interact with experts in one or the other of the four fields.

We can foresee a lot of interactive work based on the encounter during the five days of intensive work.

The Institute offered optimal conditions for this type of broad ranging interdisciplinary work in a very new area.