The many facets of the Maslov index

organized by
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Workshop Summary

The workshop was devoted to the Maslov index, the common name of a multi-faceted array of related invariants in various branches of mathematics, such as symplectic topology, algebra, dynamical systems, bounded cohomology, and mathematical physics. Rather than attack a single problem, the organizers therefore brought together specialists from the various areas, many of whom had not met before, and only knew of each other by reputation.

At the request of AIM the workshop followed the standard AIM style: we had two lectures in the morning in the main room, and two simultaneous working groups in siderooms in the afternoon. This worked very well! Étienne Ghys wrote about the meeting in general and the benefits of the AIM recipe for workshop organization in particular in an article for *Images des Mathematiques*

http://images.math.cnrs.fr/American-Institute-of-Mathematics.htmlhttp://images.math.cnrs.fr/American-Institute-of-Mathematics.html. Maxime Bourrigan's transcript of the workshop talks is available from

http://www.maths.ed.ac.uk/~aar/aimnotes.pdfhtp://www.maths.ed.ac.uk/~aar/aimnotes.pdf.

Here are some comments on the individual contributions, listed chronologically.

On Monday morning we started with Étienne Ghys' introduction to the Maslov index as the signature nonadditivity invariant, its calculation via the cohomology of the symplectic group, and the connection with hyperbolic geometry. This was followed by a lecture by Emmanuel Giroux on the application of the Maslov index and symplectic reduction to a proof of Bott periodicity in the homotopy theory of the classical groups. The afternoon discussion topics were on linear symplectic algebra led by Jean Lannes, who gave the algebraic background of the Giroux lecture, and the Nonlinear Maslov index I.

On Tuesday morning we started with Leonid Polterovich on the Maslov index in symplectic topology, with particular reference to the Gromov theory of pseudo-holomorphic curves, rigidity theorems and the group of symplectomorphisms. This was followed by Denis Auroux on the role of the Maslov index in monotone Lagrangian tori and Floer homology. The afternoon discussion topics were on signatures of fibred 4-manifolds (with a bravura performance by Auroux) and the nonlinear Maslov index II.

On Wednesday morning we started with Joseph Keller taking us through his classic 1958 paper *Corrected Bohr-Sommerfeld Quantum Conditions for Noseparable systems* providing new insights to his fundamental contribution to the Maslov index *avant la lettre*. This was followed by Claude Viterbo on the Maslov index in dynamical systems, positivity and Fourier transforms, . The afternoon discussion topics were on the various definitions of the Maslov index and the boundary of the Siegel domain.

On Thursday morning we started with Michelle Bucher on the connections between the Maslov index, quasimorphisms and bounded cohomology. This was followed by Sylvain Cappell on the application of the Maslov index to Casson's invariant for $SU(2)$-representations of fundamental groups. The afternoon session were on bounded cohomology and on the Casson invariant, to satisfy the audience's appetite for more information about the morning lectures.

On Friday morning we started with Jean Lannes on his collaboration with Jean Barges on the Maslov index from the point of view of symplectic linear algebra and the transversality defect. This was followed by Andrew Ranicki describing some number theory jump functions which arise in Maslov index calculations (slides incorporated in the Bourrigan notes). The afternoon was devoted to a problem session which was moderated by Claude Viterbo - details in the Bourrigan notes.

The participants included both seasoned and young researchers, including some graduate students from neighbourig Stanford. All came away from the meeting having learnt some of the many attractive features of the Maslov index both inside and outside their own specialty, and making new connections. Here are two particular examples.

The discussion of contact orderability and the theory of translated points of contact transformations during one of the workshop afternoon session resulted in a new collaboration between Margherita Sandon and Roger Presas one one side and Stanford participants Matthew Strom Borman and Yakov Eliashberg on the other concerning a potentially new invariant which should allow to obtain new lower bounds for the number of translated points.

During the workshop it became apparent that Michelle Bücher-Karlsson and Andrew Ranicki both had graduate students working on the signatures of surface bundle, one from the symplectic geometry point of view and one from the algebraic surgery point of view. As a direct consequence of the workshop all four will meet in Switzerland in July 2014. Étienne Ghys discovered that these questions on signatures of surface bundles are much more subtle than he previously thought and he would be delighted to collaborate!