K3: A new problem list in low-dimensional topology

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Workshop Summary

Background

Low-dimensional topology has long been a highly active field, with remarkable achievements and breakthroughs coming at a staggering pace over the last half-century. The subject, comprising knot theory and the study of manifolds of dimensions up to four, has been repeatedly transformed by new ideas coming from hyperbolic geometry, differential geometry, mathematical physics, contact and symplectic geometry, and geometric analysis, as well as advances in traditional topological methods. A fundamental contribution to the progress and unity of the field was played by two problem lists **K1** and **K2** [kirby:problems76,kirby:problems96], compiled by Rob Kirby, the second representing a significant expansion and updating of the first. These documents provided a unification of diverse ideas and stimulated and guided research in the field. It is considered a signature achievement to solve a problem from Kirby's list, and those who do so will usually point it out in their papers. With Rob Kirby providing leadership, many mathematicians contributed problems, and others detailed write-ups that provided background and inspiration for early-career mathematicians.

Since the publishing of **K2**, there have been other lists of problems in specific areas in low-dimensional topology, but none comprehensive enough to have the broad influence of the original lists. There has been a strong desire amongst leaders in the field and junior colleagues alike to produce a new list of fundamental problems. Still, the sheer magnitude of the task in the face of the explosive growth of the field makes this an intimidating prospect. The aim of the **K3** project is to create a new incarnation of these famous lists, and the AIM workshop was sought as a kick-starter event where most of the collaboration and the problem list would be shaped.

Our workshop was a bit outside the typical AIM model in which a group of mathematicians comes together to work on a particular set of problems. However, we strongly believe that asking the correct question is an essential part of research, and that a well-constructed source of problems provide structure and guidance that will immensely stimulate research in low-dimensional topology. The creative dynamic fostered by bringing together established leaders in the field along with more junior people who are bringing in fresh ideas and perspectives will yield new results, as well as novel approaches to many fundamental problems to be further explored by the larger community.

Structure of the workshop

The workshop brought together a diverse group of senior, mid- and early-career mathematicians who not only have broad mathematical knowledge, but are also leading researchers in their areas and are good at collaborating and communicating with others in their research

communities. A dedicated week-long workshop was a highly effective vehicle for getting the **K3** project up and running.

Besides the three organizers, there were 23 invited participants and five participants selected from the pool of applicants to the workshop, who were fully funded, as well as five local, partially-funded participant, and one who self-funded. A few other local participants showed up and participated in some of the discussions. To gain some initial momentum for the workshop activities, we asked the participants to review **K1** and **K2** problems in their areas of expertise and develop original problems.

The workshop ran daily from 9 am (sharp) till 5 pm, from Monday, October 30 to Friday, November 3. Many participants stayed well past 5 pm to finish work and to socialize. There were short (each 25-30 minutes) morning talks by a few colleagues on various topics to stimulate discussions:

- Monday, 10/30: Danny Calegari (U Chicago), Robert Lipshitz (U Oregon)
- Tuesday, 10/31: Dave Gabai (Princeton), Mark Lackenby (Oxford)
- Wednesday, 11/1: Ciprian Manolescu (Stanford), Laura Starkston (UC Davis)

Thursday, 11/2, and Friday, 11/3, were focused on typing up the problems and reviewing the problems in **K2** so a small percentage of still open problems from latter could be included in **K3**.

The organizers provided instructions and moderated discussions with all the participants each morning before and after the lectures and at the beginning of the afternoon session, right after the lunch break. The participants went into the break-out rooms after our gatherings in the large lecture room.

The groups divided their time between discussions of questions and actual writing of the individual problems. In the compiling of the **K3** problem list, we mirrored the division of topics from the original lists, which were more or less by dimension:

(1) Knot theory
(2) Surfaces
(3) 3-manifolds
(4) 4-manifolds
(5) Miscellaneous (typically higher dimensional)

Many subjects in low-dimensional topology have connections to many if not all of these "dimensions'. We encouraged conversations about problems that cross disciplinary borders, and to facilitate them, we varied the themes for the break-out rooms. The topics were often chosen after polling with the participants. There was much discussion and interaction between participants from different subfields; indeed, such cross-fertilization has been an essential source of new ideas and progress in the broader field.

The outcome

Before the AIM workshop, we created an Overleaf document for all the participants to collaborate on, divided into the five chapters mentioned above, and further split into individual problems the contributors could work on independently. Throughout the week, in various combinations, the participants discussed their proposed problems in groups until they agreed on whether or not they should make it to the list and in which forms. The discussions provided new perspectives and directions for the proposed problems, which often changed even the formulation of the questions. Once there was consensus, the problem was typed up and included in the Overleaf document.

The problems were discussed in various forms: Some are well-known open problems in the field, so the discussions mainly focused on possible connections and best formulations of the problem. Sometimes, it was an expert who typed up the problem; sometimes, it was scribed by a participant in the group discussion who wanted to learn more about that topic. There were several highly original problems that were brought in by the participants. Some others were proposed to the organizers by other colleagues in the field, and the AIM workshop participants discussed them to provide feedback—and to decide whether or not they would make it to the **K3** list.

Before the workshop, we had also set up EditFlow, which is used as an editorial interface by many mathematics journals, for further thorough evaluation of the K3 problems post-workshop. When the problems were deemed to reach their final form on the Overleaf document (at which stage they could be viewed by all the participants and could be commented on), the proposer or scribe shared it with another participant for final review and after their "seconding" the problem submission, the problem was submitted to EditFlow.

To facilitate the group structures and the main document's writing, we recruited 12 "chapter editors" among the participants at different career levels, who ensured that the discussed problems would be typed up properly and submitted to EditFlow. The chapter editors will continue to serve as associate editors for the problem list. They helped tremendously with moderating some of the group discussions and setting some standards and uniformity for the problems on the list. They are responsible for maintaining and editing the problems in a particular section.

In Kirby's earlier compilations, the initial list was greatly expanded and refined by consultations with a large number of mathematicians who had not been present for the creation of the initial list. Our AIM workshop participants also serve as the delegates of their larger research communities and consult their colleagues at any stage–before, during or after the workshop–as they deem necessary.

Our main goal was that by the end of the week, there would be a substantial number of problems collected and written in something close to the final form. We had around 230 completed problems at the end of the week, roughly the half of the number of problems we expect to have on the final list of problems in **K3**. The **K1** and **K2** were published in conference proceedings, which brings the advantage of having a document of record to which authors can point when solving a problem. We have similar plans for **K3**, which we'll finalize in communication with our chapter editors.

In our view, the AIM workshop was wildly successful. The participants brought in many new ideas, discussed them intensively, and wrote them up with care. Not just us, but many participants too, were amazed by the sheer amount of new ideas and interesting questions they learned about in one week. We have strong hopes that **K3**, when made publicly available, will play the same role as the previous lists in stimulating new research across the now mature and broad field of low-dimensional topology.

Bibliography

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