Descriptive inner model theory

organized by
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Workshop Summary

The main goal of the workshop was to present and discuss recent advances concerning the core model induction technique, the analysis of the hereditarily ordinal definable sets in models of AD, and inner models with long extenders. There was a workshop the following week, June 9-13, 2014, at UC Berkeley, also organized by Schindler and Steel, and devoted to the same topics. Most of the AIM workshop participants attended this follow-on workshop as well.

Our mornings at AIM were filled with talks by Itay Neeman, Hugh Woodin, Grigor Sargsyan, and Nam Trang.

Neeman reported on results with Steel which produce optimal consistency strength lower bounds from failures of square at a Woodin cardinal, in the presence of UBH (the unique branch hypothesis for iteration trees on V using short extenders only). The talks were based on a paper, "Equiconsistencies at the level of subcompact cardinals", that was made available to participants. Extending this work, Neeman and Steel more recently developed a scenario for obtaining an inner model with a cardinal κ which is κ^{+n}–supercompact, $n < \omega$, from the same hypotheses. The model in question is constructed from a sequence of short and long extenders, and in a series of talks at the follow-on Berkeley workshop, Steel presented key aspects of the relevant fine structure theory developed by Neeman and him. These talks were based on a set of notes titled "Plus-one premice", again made available to workshop participants.

Woodin had earlier and independently started developing a fine structure theory for mice with long extenders. His preprint on it had been made available to the participants before the start of the meeting, and in his talks he went through key details of his approach. He also gave a follow–up talk during the week after the AIM meeting.

Sargsyan's talk presented the basic theory of HOD mice, structures needed for analyzing the HOD of a model of determinacy. The current frontier is around the hypothesis that the largest element of the Solovay sequence below Θ is a Suslin cardinal, and in follow–up talks during the next week Sargsyan reported on how to organize HOD mice at this level.

Trang showed us details about the core model induction concerning an application to guessing models, introducing many of the relevant concepts and structures. His talk gave a very good picture of how the core model induction is organized as a nested induction. He also continued the week after the AIM workshop.

New papers covering the work described by Sargsyan and Trang were also made available to workshop participants.

The workshop afternoons were devoted to problem sessions and working group activities.

We went through the list of problems which was established during an earlier AIM meeting, "Recent advances in core model theory," organized by E. Schimmerling and J. Steel, Dec 13-17, 2004. It was an exciting experience to discuss almost 10 years later the status of those problems. We also produced a new list containing sixteen problems which we hope will inspire future research in areas tightly connected with the topic of our workshop.

Concerning the group activities, the organizers were impressed by how productive they turned out to be.

Working group #1 ("HOD mice") went through aspects of the core model induction, with an eye towards identifying which details of the induction are hypothesis–dependent, and which ones are general technical issues. The example the group focussed on was the failure of square at a singular or measurable cardinal. A technical issue that was identified is the need for an "absolute" version of the Lp(–) operation, as this model is computed in several different universes, so that the agreement between those different versions is crucial. This issue is what stops the induction short of reaching the goal of getting a model of $\mathrm{AD}_{\mathbb{R}}$ and Θ regular. Another topic was identifying the large cardinal assumptions a HOD mouse must satisfy in order to reach certain determinacy levels.

Working group #2 ("Square in long extender models") went through the Schimmerling–Zeman proof of square in short extender models below subcompact cardinals. They were greatly aided in doing this by Martin Zeman, who joined the group periodically. The goal was to generalize the proof to fine structural models with long extenders. They did not get far enough during the workshop that they could do that, but they did get closer. Several participants plan to continue working on this project in the immediate future.

Working group #3 ("HOD in $L[x]$") looked at ways of approaching simplifications of the problem of analyzing HOD of $L[x]$, where $x \in \mathbb{R}$, so that one would only need to prove comparison in countably many steps for models which are already known to be iterates of M_1. They developed promising ideas for how to do this, ideas that now will have to be checked.

Working group #4 ("Correctness of K") discussed issues concerning the amount of correctness of the core model K. As a new result, they proved that if every real has a sharp and every subset of ω_1 is constructible from a real, then there is an inner model with a Woodin cardinal, and they tried (so far, unsuccessfully) to expand their result to show that if every real has a sharp and u_2, the second uniform indiscernible, is equal to $\aleph_2$, then there is an inner model with a Woodin cardinal. They also developed a scenario for proving that the core model below 2 Woodin cardinals is Σ^1_4 correct, provided V is closed under the $M_1^{\#}$ operation.

Working group #5 ("Dual covering for K") obtained very interesting results on representing sets of ordinals as unions of sets in the core model K. Let a set of ordinals X be called relevant if $\omega_2 \subset X$ and X is closed under countable suprema. They proved that (1) it is consistent relative to the existence of ω_2 measurable cardinals that there are stationarily many relevant sets in $\mathcal{P}_{\omega_3}(\omega_4)$ which are not the union of $\aleph_1$ sets in K, and (2) if there is no inner model with ω_2 measurable cardinals, then for every cardinal $\kappa < \aleph_{\omega_2}$, almost every relevant subset of κ is a union of $\aleph_1$ sets in K.

The workshop brought together most of the world's experts in descriptive inner model theory. Many post-docs and advanced graduate students attended as well. The AIM system of talks in the mornings and discussions and working groups in the afternoon worked very

well: we covered the main ideas in some substantial new papers, while keeping everyone actively involved. The subsequent Berkeley week built on this success. We believe that these meetings were a great service to the community of researchers working in the area of Descriptive Inner Model Theory.