CLASSIFICATION OF GROUP VON NEUMANN ALGEBRAS

organized by

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Workshop Summary

The workshop brought together a diverse group of 21 researchers with expertise in von Neumann algebras, C*-algebras, geometric group theory, Lie groups and lattices, and ergodic theory in order to make progress on central questions in the classification theory of group and group-action von Neumann algebras.

Workshop activities were organized as follows. Each day there were two talks in the morning. On the first day this was followed in the afternoon by an open problems session. On subsequent days the workshop participants split into groups which were tasked with exploring specific questions that were raised during the problems session. Over 30 problems were generated and discussed during the open problems session and several more that arose during group discussions over the week were also added to the problems list.

The speakers for the morning talks presented on a variety of topics and perspectives which are regarded as potential sources of new ideas in the classification of von Neumann algebras. Many topics were also chosen in order to provide background and to help facilitate discussions among researchers of differing expertise. On the first morning Thomas Sinclair gave an introductory talk on group and group-action von Neumann algebras and Sorin Popa gave an overview of the major open problems remaining in the classification of group and group-action von Neumann algebras. The second morning session's talks were given by Narutaka Ozawa on the computer-assisted proof of property (T) for $Aut(\mathbb{F}_5)$ and by Alex Furman on an overview of measured group theory and a possible measured group theory analog of hyperbolicity. On the third morning Rémi Boutonnet spoke on the use of group boundary techniques in von Neumann algebras and Denis Osin spoke on acylindrical hyperbolicity. The fourth morning session had talks given by Sven Raum on the classification of reduced group C*-algebras including C*-superrigidity of torsion-free, finitely generated, 2-step nilpotent groups and Cyril Houdayer on central sequences and fullness in type III factors. On the fifth and final day of the workshop the morning's talks were given by Ana Khukhro on the quasi-isometric classification of box spaces with applications to expanders and by José Carrión on quasi-diagonality and recent advances in the classification of amenable C*-algebras.

We now turn to a summary of the activities of the various groups which met for afternoon problems solving sessions over the week.

• **Ultrapowers of von Neumann Algebras.** Several questions on the classification of ultrapowers were considered. It was observed that the construction of uncountably many families of McDuff II_1 factors with distinct ultrapowers could be generalized to the the same result for type III_λ factors for each $\lambda \in (0,1)$. Discussion then turned to question of constructing a solid II_1 factor which is not "ω-solid", i.e., a II_1 factor M for which the commutant in M of any noninjective subfactor is never diffuse but for which there is a

noninjective subfactor of M with diffuse commutant in an ultrapower of M. Several potential constructions were proposed.

- **W*-superrigidity of property (T) groups.** Recall that a countable discrete group Γ is said to be W*-superrigid if it is the unique discrete group up to isomorphism which generates its group von Neumann algebra $L(\Gamma)$. The goal of the group was to construct a non-W*-superrigid property (T) group in order to provide a counterexample to Connes' rigidity conjecture or find the first example of a W*-superrigid group with property (T). Several possible constructions of such groups were discussed, including a proposed family of wreath product-like groups with property (T). Such groups would provide interesting test cases as wreath product groups are known to never be W*-superrigid, while certain generalized wreath product groups are W*-superrigid.

- **C*-superrigidity of countable discrete groups.** It is conjectured that every countable discrete group is the unique discrete group generating its reduced group C*-algebra. In light of recent work establishing this conjecture for torsion-free 2-step nilpotent groups, the case of 2-step solvable groups was discussed. One particular question discussed was whether the wreath product groups $\mathbb{Z} \wr \mathbb{Z}$ and $\mathbb{Z}^2 \wr \mathbb{Z}$ generate nonisomorphic group C*-algebras.

- **Discrete embeddings of groups in the unitary groups of II$_1$ factors.** Specifically the group was interested in the problem of whether every discrete subgroup of the unitary group of the hyperfinite II$_1$ factor is amenable. It was observed during group discussion that any discrete subgroup of the unitary group of the hyperfinite II$_1$ factor has Haagerup's approximation property. It was previously known that no discrete subgroup of the unitary group of the hyperfinite II$_1$ factor has property (T). The question of whether there is a nonabelian free subgroup remains open. In this direction the group also discussed possible constructions which should randomly generate a discrete free subgroup of the hyperfinite countable equivalence relation of type II$_1$.

- **Uniformity of non-Gamma factors.** The question of whether the negation of property Gamma is witnessed by a uniform spectral gap estimate for almost central sequences was discussed as well as the natural variant for full groups of countable Borel equivalence relations of type II$_1$. It was proposed that constructions involving groups of arbitrarily large Tarksi number would be good candidates to explore such behavior as large Tarski number implies uniformly small spectral gap over all generating sets of the group of a fixed finite size.

- **Von Neumann algebras of groups with positive first L^2-Betti number.** Questions around the structure of group von Neumann algebras of groups with positive first L^2-Betti number were discussed. A conjecture of Peterson and Thom states that a group with positive first L^2-Betti number cannot be generated by a pair of amenable II$_1$ factors with diffuse intersection. Amenability should be thought of in this context as a 1-cohomology vanishing condition to be played against the nontrivial 1-cohomology of the generating group. Using property (T) in place of amenability it was proved in discussion that the von Neumann algebra of a group with positive first L^2-Betti number cannot be generated by two irreducible property (T) subfactors with diffuse intersection.