Boundaries of groups

organized by
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Workshop Summary

Overview

We were pleased to bring together researchers from various sub-fields of geometric group theory, topology, and dynamics to collaborate on understanding boundaries of groups. We feel that we had a very successful workshop, with a lot of discussion and collaboration. One of the ways that discussion started was that the speakers (with very short notice) gave talks which were accessible to people in other sub-fields. Since their talks were central to the direction of the conference, we briefly outline them below in section . The general plan of the conference was that of a typical AIM conference: two talks each morning, problem session the first afternoon and discussion sections on specific problems the rest of the afternoons.

Morning talks

Topics for the talks were selected to be of general interest, and at an introductory level. Overview talks were given on the following topics, listed below chronologically:

- Ends of groups and spaces (Craig Guilbault)
- Analysis on boundaries of groups (Mario Bonk)
- Various boundaries for CAT(0) groups (Ruth Charney)
- Conformal dimension and applications (Marc Bourdon)
- Hierarchically hyperbolic spaces (Mark Hagen)
- Boundaries and rigidity (Misha Kapovich)
- Roller boundary of cube complexes (Talia Fernos)
- Relatively hyperbolic groups (Jason Manning)
- Dynamical properties of boundaries (Peter Haissinsky)
- L^2-cohomology of groups (Kevin Schreve)

Discussions!

We feel that this was probably the most important and productive part of the conference. Topics for the discussion sections were chosen to feature a mix of open-ended questions, as well as more concrete problems. Listed below are (some of) the topics discussed, as well as the progress made in the individual groups.

1. Is there an algebraic characterization of when the boundary of a CAT(0) group is locally connected? The group found a few existing theorems in the literature which give algebraic sufficient conditions that imply that the boundary is not locally connected and explored a few examples. The answer is known for right-angled Artin groups, and mostly known for right-angled Coxeter groups. A complete answer is also known in the case of groups acting on CAT(0) spaces with isolated flats. There wasn't much progress on a generalization to a wider class.

2. If a group G is relatively hyperbolic, relative to a subgroup H that is semi-stable, must G also be semi-stable? The group obtained an outline for a proof under a slightly stronger hypothesis – they had to assume that the subgroup H can be Z-compactified by a locally connected set (this implies H is semi-stable).

3. Develop rigidity results for the Gromov-Thurston negatively curved manifolds. The group reported good progress. They have an argument to show that *all* GT manifolds are not homeomorphic to locally symmetric manifolds. They also made progress towards showing that any QI from the universal cover of a GT manifold to itself must coarsely preserve the collection of lifts of branching loci.

4. The "right" definition of EZ-Structure for groups with torsion. The group first discussed the appropriate definition for a group to admit an EZ-structure if it has torsion, defined in terms of Z-compactifications of normalizers of finite subgroups of the group. We then discussed what known examples would satisfy this new definition (CAT(0) and delta-hyperbolic groups and products of groups admitting EZ-structures), and the possibility of obtaining a geometric proof of the Novikov Conjecture for groups admitting these types of EZ-structures.

5. "Drilling" hyperbolic groups with S^2 boundary. This group discussed the analog of drilling a hyperbolic 3-manifold along an embedded geodesic in the group-theoretic setting: Can one "drill" a hyperbolic group with two-sphere boundary along a group element to obtain a relatively hyperbolic group with 2-sphere Bowditch boundary and $\mathbb{Z}^2$ peripheral subgroups? We made some progress on understanding when and how this happens and have plans (including a SQuare) to work on this problem further. We also discussed the difference between a Sierpinski carpet which is the Bowditch boundary of a relatively hyperbolic group with $\mathbb{Z}^2$ peripheral groups and one which is the boundary of a hyperbolic group.

6. Path Connectedness and Divergence. This group considered the questions: Under what conditions are two points in the contracting boundary of a CAT(0) space connected by a path? Can path connectedness be characterized by divergence properties? Consider the full spectrum of divergence functions for geodesics in a CAT(0) group (rather than just the maximal divergence). What can the spectrum look like? Use it to distinguish quasi-isometry types of specific groups. The second question morphed on the last day into an ongoing project addressing the following question: Given a group G acting geometrically on a CAT(0) space X, is it true that either G is not relatively hyperbolic, or is hyperbolic relative to a collection of subgroups, each of which has strictly lower divergence than G?