

# GEODESICS

organized by  
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## Workshop Summary

The joint workshop on Geodesics sponsored by the American Institute of Mathematics (AIM) and the Chern Institute of Mathematics (CIM) was held during August 23-27, 2010, at the Chern Institute of Mathematics, Nankai University, Tianjin China. This workshop was a great success. 35 mathematicians participated in it. The participants came from US, UK, Germany, Italy, Canada, France, Korea, and China.

The workshop was devoted to the study of the behavior of geodesics in the large. Although this is an old subject, with important contributions first made by J. Hadamard and H. Poincar, many of the fundamental problems are still open.

The study of geodesics is a central part of Riemannian geometry, of the calculus of variations and of Hamiltonian dynamics. Beyond that it is related to a remarkably large number of mathematical and non-mathematical fields ranging from the study of PDEs to number theory and from transport problems to general relativity. So it is no surprise that often progress in geodesic problems is achieved by combining ideas and methods from different areas.

Here is a non-exhaustive list of topics that have been discussed in the workshop.

- Closed geodesics and geodesic networks.
- Morse theory on loop spaces.
- Statistical properties of geodesic flows, dynamical zeta-function.
- Rigidity and geodesics.
- Symplectic geometry and J-holomorphic curves applied to geodesics.

The workshop was different from typical conferences in some regards. Participants have been invited to suggest open problems and questions before the workshop, and these have been posted on the workshop website. There were 13 talks given in the morning, and free discussion in the four afternoons during the workshop.

The workshop has stimulated exchange and cooperation between researchers from diverse mathematical fields with interest in geodesics. The organizers received much positive feedback on the quality and accessibility of the talks. Ample time was left in the program for discussion among participants, and it was generally agreed that many new bridges were built linking the different groups and that new collaborations were begun (please see the attachment in the next page).

List of speakers and the title (in chronological order):

- (1) M. Pollicott: Zeta functions for geodesic flows
- (2) Y. Long: Multiple closed geodesics on compact simply connected manifolds
- (3) R. Rotman: Lengths of geodesics on closed Riemannian manifolds
- (4) N. Hingston: Loop products and resonance for loop homology on spheres
- (5) S. Sabourau: Curvature-free inequalities on surfaces and the minimax principle on the one-cycle space
- (6) K. Burns: Ergodicity of the Weil-Petersson geodesic flow
- (7) G. Paternain: Transparent connections
- (8) B. Schmidt: Closed geodesics without proper chords
- (9) V. Matveev: Weyl-Ehlers Problem
- (10) G. Knieper: New results on noncompact harmonic manifolds
- (11) V. Bangert: Cylinders without conjugate points
- (12) M. Schwarz: Floer homologies and loop space topology
- (13) W.-K. Ku: A dense G $\delta$  set of Riemannian metrics without the finite blocking property

### **Report on the discussion groups.**

The discussions in the afternoon sessions were stimulated by the morning lectures. In particular, there were discussion groups on

- Closed geodesics on Riemannian and Finsler manifolds
- Geodesic networks and systolic inequalities
- Riemannian metrics without conjugate points on cylinders and planes
- Relations between the closed geodesic problem and zeta functions
- Partially hyperbolic geodesic flows
- Geodesics on compact surfaces with nonpositive Gaussian curvature

Some of the groups met only once, others twice. The discussions evolved along the list of problems compiled beforehand, and sometimes added new problems to this list. A large part of the discussions concerned old and new ideas how to attack these problems. Some progress was made on planes without conjugate points and on the question (due to M. Herman) if there are partially hyperbolic geodesic flows that are not Anosov. The discussion groups led to plans for collaborations between some of the participants, in particular but not exclusively in the areas where some progress was made during the workshop.