

FOURIER ANALYTIC METHODS IN CONVEX GEOMETRY

organized by

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Workshop Summary

The Fourier analytic approach to sections and projections of convex bodies has recently been developed and has led to several important results including a solution to the Busemann-Petty problem on sections of convex bodies. The idea of this approach is to express certain geometric parameters of convex bodies in terms of the Fourier transform and then apply methods of harmonic analysis to solve geometric problems.

The workshop had several goals. First, we wanted to clearly identify the direction of research, its connections with neighboring areas of mathematics and the main open problems. This was done through a series of introductory lectures, each of which included an overview of a certain part of the area and connected open problems. Koldobsky presented an introductory talk on Fourier methods in convex geometry. Rolf Schneider of the University of Freiburg surveyed applications of spherical harmonics to convex geometry. Richard Gardner from Western Washington University gave an excellent account of duality in convex geometry, problems of geometric tomography and related integral transforms. Hermann König from University of Kiel spoke about extremal problems in the theory of sections of convex bodies. Emanuel Milman (IAS) characterized different generalizations of intersection bodies - the main concept in the area - and related them to open problems of the Busemann-Petty type. Roman Vershynin (UC-Davis) explained the connections between convex geometry and the theory of eigenvalues of random matrices. Sinai Robins (Temple University) outlined the connections between number theory, Fourier analysis and convexity. Elisabeth Werner (Case Western) spoke about problems related to the surface area of convex bodies. Gideon Schechtman (Weizmann Institute of Science) surveyed connections with the local theory of Banach spaces, particularly, the Dvoretzky theorem. Wolfgang Weil (University of Karlsruhe) presented several results on identifying convex bodies from different properties of their sections and projections. These talks took place in the morning and, besides introducing different aspects of the subject, greatly influenced the discussions in the afternoon.

The afternoon sessions on Monday and Tuesday started with moderated problem sessions, where the participants were asked to propose open problem and discuss them. These sessions were successful, as many problems were identified and the actual work on them started. Richard Gardner proposed the problem of whether every convex body can be represented as the positivity set of the sum of four functions, each of which is constant on parallel lines in one direction. This is a very important problem in the theory of x-rays. Koldobsky asked whether intersection and polar projection bodies are isomorphically equivalent, which would contribute to understanding of the duality between sections and projections. Hermann König asked several question about extremal sections and slabs in convex bodies. Rolf Schneider raised a question of whether intersection bodies of convex bodies that are not

polar projection bodies form a large class (in the Baire sense) in the set of all convex bodies. There were many other questions that were accompanied by lively discussions.

The afternoon sessions also included work in groups. A group led by Rolf Schneider worked on the theory of valuations, Richard Gardner formed a group that worked on geometric tomography, in particular the problem mentioned above. Hermann Koenig led the discussion of the problem of finding the extremal slabs in the cube. Gideon Schechtman and Stanislaw Szarek led the group discussing several problems of the isomorphic theory of convex bodies. These discussions will definitely lead to new collaborations and, hopefully, to solutions of some of the problems.

The participants included a quite big group of postdocs and graduate students, who benefited from the introductory lectures and opportunities to participate in working groups. Several sessions were conducted where students were able to get answers to their questions from the lectures. Two such sessions were led by Koldobsky and one by Richard Gardner.

Finally, we accomplished our main goals. This workshop was a very good starting point for our NSF FRG research project “Fourier analytic and probabilistic methods in geometric functional analysis and convexity” that was funded in June 2007.

As we mentioned above we were able to gather the list of extremely interesting questions in the area. We hope that these questions will attract wide mathematical community and many problems will be solved. The solutions to any of those problems would lead to a significant progress in the area.

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