First passage percolation and related models

organized by
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Workshop Summary

1. Overview and Workshop Aim

Stochastic growth models have been studied since the 1960's, and have their roots in theoretical physics and biology. Such systems address the behavior of growing interfaces (in first-passage percolation (FPP) and competition models), the spread of bacterial colonies (the Eden model and variants), and the fluctuations of long chemical chains in a random potential (polymer models). Predictions from physics indicate that in many of these cases, fluctuations should scale sub-linearly with limiting laws that deviate from the standard Gaussian (for instance, which relate to the Tracy-Widom distribution from random matrix theory).

The aim of this workshop was to bring together senior/junior researchers and graduate students who work on general questions in growth models including FPP, polymer models, and the contact process. Our agenda was twofold: first we aimed to consolidate the current knowledge on these models away from the few exactly solvable cases. Second, we put forth a research agenda for the future of the area. In particular, the topics discussed included questions of existence and properties of the asymptotic shape, shape fluctuations on the approach to the limit, geodesics and Busemann functions, scaling exponents, and coexistence in growth models.

There were a number of survey talks representing the current state of the field; these discussed many of the important outstanding problems in the subject and the most successful modern proof techniques. On the first afternoon, there was an extensive problem session, moderated by Firas Rassoul-Agha, which helped the organizers to select a list of problems for group work later in the week. The participants initially suggested 25 research problems. This list is available at the AIM website and is included in the survey written by the organizers.

In total, the workshop had 27 participants from the US, Canada, France, Japan, and Brazil. These included some of the senior experts in the subject as well as many young researchers and graduate students.

2. Summary of the Talks

On the first day, we had talks by Jack Hanson and Michael Damron. Jack spoke on basic methods and the best well-known results on the time constant and the limit shape in FPP. Michael spoke about the variance of the passage time and the wandering exponent. On the second day, we had talks by Chuck Newman and Chris Hoffman. Chuck beautifully outlined a heuristic argument that connects the wandering exponent to the (non-)existence of geodesic lines. Chris's talk was about geodesics and the connections to Busemann functions. He

surveyed his past results on existence of multiple geodesic rays and explained the directional results of Damron-Hanson.

On the third day, we had talks from Régine Marchand on the current state of the art of Richardson-type competition models and by Timo Seppäläinen on variational formulas for last-passage percolation with general weights. Timo's talk was complemented by Firas Rassoul-Agha's talk on the fourth day, where the connection between the variational formulas, co-cycles and geodesic rays was explored. On the same day, we also had a talk from Jean-Christophe Mourrat on certain properties of polymer models and on homogenization of equations of divergence form. Finally, on the last day of the workshop we had a talk by Phil Sosoe on sub-linear variance and concentration of the passage time and a talk by Daniel Ahlberg on large deviation estimates in FPP.

Taken together, these high quality lectures gave an excellent overview of the important techniques, recent developments, and future challenges in first-passage percolation and related models. A highlight was the use of the blackboard by all lecturers and the collaborative environment during the presentations.

3. Summary of Working Groups

From the initial list of 25 open questions, the organizers selected 12 problems for further study on the second day of the workshop, and these formed the basis of group research for the rest of the week.

Several ideas emerged from these discussions including partial progress in several groups. We list a few examples of those here (the list is non-exhaustive). One group tried to relate the scaling relation $2\chi = \xi$ to the linear growth of the variance of the Busemann function in two dimensions. If one could show that successive increments of the Busemann function were positively/negatively correlated, one may hope to obtain sharp inequalities for the exponents. Another group seemed to have solved their chosen open problem. They obtained a complete solution for the fluctuations of the passage time along a reinforced axis, in an FPP model with two different passage time distributions.

Some interesting observations about constructions for the time constant in high dimensions were given. Another problem considered was that of determining the empirical measure of the environment seen by a geodesic. Some preliminary results and simulations were obtained by the group, and the report was promising. This problem also emerged in the discussion of the differentiability in p of the time constant in FPP with Bernoulli(p) weights. In this group, the participants identified several cases where the differentiability was already known and devised some new approaches that may allow substantial progress in this direction. The problem of proving divergence of fluctuations on the $2d$-torus was considered. The group in question provided a good explanation of why the Newman-Piza martingale method only provides a constant lower bound. Here, it seems that a new idea will be needed to push the bounds past this bottleneck. Finally, the 50-year-old open problem of strict convexity of the limit shape was considered in the last day by one group, but the problem resisted intact.

4. Concluding remarks

Based on the initial feedback from participants (especially the younger researchers) and the friendly working atmosphere during the week, we believe the workshop was a true success. It is expected that a number of new collaborations may result from the workshop and at least a few papers should grow out of the problems considered.