Computable stability theory

organized by
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Workshop Summary

The workshop on Computable Stability Theory was planned to bring together researchers from computability who work in computable model theory and researchers from model theory. In its formative stages, much of the development of model theory was motivated by questions of computability of structures or theories, so model theory and computable model theory were two sides of the same developing theory. Over the course of the last few decades, the topics have grown rather separate. Indeed, it was a goal of the workshop to bridge the gap that had formed between these two fields. Recently, computable model theory has begun to form several connections with modern model theory. These connections use structural results from stability theory to bound computability of structures or models, and in many cases have also led to a better understanding of the structure of the models. The workshop focussed especially on the following topics.

- Automatic quantifier elimination
- Fraïssé limits and related constructions
- Complexity of embeddings

There were two talks each morning. These were designed to introduce notions and techniques from computability, or model theory, to participants from the other group. In his introduction, Andrews got the whole group to participate in a preliminary discussion of some of the topics to be covered during the week. Antonio Montalbán introduced priority constructions, for the benefit of the model theorists. The treatment was interesting also to the computability people. It indicated clearly the role of back-and-forth relations. Dave Marker gave an overview of stability, for computability theorists. Chris Laskowski described a very successful point of contact between model theory and computable model theory—the result of Goncharov, Harizanov, Laskowski, Lempp, and McCoy, saying that for a trivial strongly minimal theory, if one model is computable, then all models have Δ^0_3 copies. The question came from computable model theory, but the answer involved a new way to produce model complete theories. This result has led to a body of further results, by Dolich and others.

Julia Knight recalled a statement of Kreisel that a general result was like a loaf of bread that needed jam—interesting corollaries or examples. She described a result of Goncharov-Nurtazin, and Harrington, characterizing the elementary first order theories that have decidable prime models. In his short paper, Harrington provided lots of jam—the fact that for a decidable $\aleph_1$-categorical theory, all models have decidable copies, a result of Rabin on algebraic closures, and an analogous result on differential closures. Russell Miller gave conditions, involving a set of finitely generated structures and embeddings, guaranteeing that an uncountable structure is $\infty\omega$-equivalent to a computable one. John Baldwin, introducing the topic of generalized Fraïssé constructions, got the group to play a game, putting different

conditions on the "age" in order to arrive at different limit structures. Barbara Csima, put conditions on the age to get a computable limit structure. She also gave some examples. Noam Greenberg, described a notion of computability, "α-recursion", for $\alpha = \aleph_1$, which, with some set-theoretic assumptions, makes the real number field a computable structure. Karen Lange gave background on real closed fields, leading to a result characterizing the recursively saturated ones.

On Monday afternoon, there was a big problem session, moderated by Dave Marker, with problems suggested by many participants. On the other afternoons, participants chose from a small set of problems and broke into groups to work on their choice. There was a fresh set of problems each day. There were some successes. We are extremely pleased that several of the discussions have continued beyond the workshop.

We summarize the topics and discussions of some of the larger groups. One large group worked for several afternoons on the topic of automatic quantifier elimination. Though no new results were proved during the workshop, a new understanding of the obstructions resulted. Work on this problem has continued since the workshop, most notably by Dolich, who now claims that every trivial, uncountably categorical theory has a finite bound on the quantifier complexity of the elementary diagram of any model of the theory.

One group worked on trying to give proofs in second order arithmetic of theorems asserting the existence of long, independent sequences in e.g., sufficiently saturated models of simple theories. The standard proofs of these results usually employ the Erdös-Rado theorem, which cannot be formalized within this logic.

One group discussed the connections between classification theory and problems arising from uncountable computable model theory. In this case, the main proposed question was fully answered by simply bridging the language barrier that has developed between the fields. Specifically, by bringing together experts in both fields, it was noted that deep theorems of Shelah on classifiable theories (in particular an algebraic characterization of models of $\aleph_0$-stable, non-multidimensional theories with NDOP) could be massaged into a proof of the proposed question. These connections can clearly be extended to answer other similar problems in uncountable computable model theory.

Another successful group worked on a problem regarding the Scott sets of recursively saturated real closed field. This led to an answer of a question of Marker's, which Karen Lange then presented the solution on Thursday.

Lastly, we mention a group which worked on the lengths of generalized power series required in the Mourgues and Ressayre embedding, which was solved at the workshop (though the proof fell apart later, but was revived by work of Knight, Lange, and Starchenko).

In choosing participants, the organizers chose people from model theory who seemed open to interaction with people from computability, and vice versa. The group was, in fact, extremely harmonious. With one exception, the working groups in the afternoons all included at least one model theorist and at least one computability theorist. The discussions that took place before the morning talks, during coffee, and at lunch, and in the shuttle on the way to the airport, also cut across the specialties.

After the workshop ended, the organizers received many e-mail messages indicating that the participants enjoyed the workshop, and some of them are continuing to think about the problems. Marker continued the discussion with Dolich, Lange, and Knight. Marker's question about whether every Scott set was the set of reals coded in a recursively saturated

real closed field was answered positively during the workshop. The question is related to the longstanding open problem of whether every Scott set is the Scott set of a nonstandard model of PA. Knight asked whether every Scott set is represented by a recursively saturated model of Presburger arithmetic—the theory of the integers with $+$ and $<$. Without the ordering, this is an old, relatively easy, result. With the ordering, Marker found a positive solution.

The organizers were very happy with the interactions at the meeting, the number of questions answered, the number of projects begun, and especially that the workshop has spurred further interest and continued collaboration across disciplines.