CANONICAL REFERENCES

This document is a compilation of suggested canonical references for various topics related to the moduli space of curves. It was produced as part of the activities during the Topology and Geometry of the Moduli space of Curves workshop in March 2005; it is a component of a larger resource site at http://www.aimath.org/WWN/modspacecurves/

This document is meant to serve both as a guide to new people entering the field and also as an aid for those coming from either an algebraic geometry or a topological background and wishing to learn something about the other side. The content here is derived from material contributed by participants prior to the workshop.

Please send comments, suggestions, corrections, etc. to either

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0.1. **Introductory material on the moduli space of curves.** As a point of entry, there is the introductory book by Harris and Morrison, [HM98]. Moving towards the research front, two excellent expository papers on this material are those by Vakil [Vak03] and Corti [Cor97]. Further details can be found in the survey papers of Faber and Looijenga [FL99] and Hain and Looijenga [HL97]. Those interested in the construction of the moduli space should see [Edi00], which contains an excellent introduction to both Deligne-Mumford stacks and geometric invariant theory, and also constructs the moduli stack and moduli space of curves.

0.2. **Introductory material on Gromov-Witten Theory.** The standard reference here is Fulton and Pandharipande's notes [FP97]. However, it might also be worth looking at Kock and Vainsencher [KV], which presents a more gentle introduction, covering the moduli space of maps, Gromov-Witten invariants and quantum cohomology.

0.3. **Introductory material on intersection cohomology.** The book by Kirwan [Kir88] covers this subject very nicely, although the original papers by Goresky and Macpherson [GM80, GM83] are definitely still worth reading.

0.4. **Tautological rings.** The point of view of tautological rings as minimal subalgebras satisfying certain conditions is explained in [Vak03], [FP00], [GV03], [FP03], [GP03]. The introductions and selected parts of these articles are fairly accessible.

0.5. **Admissible covers.** These are compactifications of the Hurwitz scheme that are smooth and very important for computations of intersection numbers. This is covered in [HM98], but the "stacky" point of view, as presented in [ACV03] is critical because it produces a smooth proper moduli stack.

0.6. **Artin stacks.** A reference for Chow groups with integral coefficients on Deligne-Mumford stacks is Andrew Kresch's Inventiones paper "Cycle groups for Artin stacks" [Kre99].

0.7. **Localization.** By considering the moduli space of stable maps (of relative stable maps) from a curve of genus g to the projective line, one can use (virtual) localization techniques to get relations in the tautological ring of the moduli space of curves (or pointed curves). The principal references for this method are [GV03] and [Kat03].

0.8. **Infinite loops spaces and topological tools.** The topological approach to understanding the moduli space of curves is partly built upon the language of basic stable homotopy and infinite loop spaces, so it is probably wise to know at least a little about the language of infinite loop spaces. The book of Adams [Ada78] is highly readable and its first few chapters are certainly the place to begin. For the relation between monoidal categories and infinite loop spaces, Segal's original two papers [Seg68] and [Seg74] make for good reading, and May's paper [May74] states the theorems of the subject nicely.

0.9. **The Madsen-Weiss-Tillmann story.** Tillmann's ICM paper [Til02] provides a nice overview of the topological approach to the Mumford conjecture. A more technical discussion in contained in [MT01].

0.10. **Connections to string topology.** Much of the story here is currently unfolding, but perhaps a good place to read about what we do know is the notes of Cohen and Voronov [CV05].

0.11. **Graph Homology.** Kevin Costello highly recommends Kontsevich's paper [Kon94] as an excellent source of ideas about graph homology and its relation to the moduli space of curves (although proofs and details are somewhat lacking).

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