Dynamics of the Weil-Petersson geodesic flow

organized by
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Workshop Summary

The focus of this workshop was the study of the Weil-Petersson metric on the Teichmuller space and moduli space of surfaces of genus g with n punctures and its recent generalization arising from thermodynamic formalism to character varieties of representations of surface groups into higher rank Lie groups. Moduli spaces can be studied from a number of different viewpoints including complex analysis, differential geometry, dynamics and metric geometry. The Weil-Petersson metric is a natural metric on the space with deep connections to hyperbolic geometry. The study of this metric has interested researchers since Weil's original observations. The workshop brought together mathematicians with a broad array of interests in various aspects of Teichmuller space, moduli space and their generalizations to higher rank representations.

The workshop followed the general AIM format with two lectures each morning followed by a general session on the first afternoon, and smaller groups focusing on specific problems on the subsequent afternoons. Scott Wolpert gave two background lectures focusing mainly on differential geometric aspects of the metric, including expansions for the metric, its Levi-Civita connection and Riemann curvature tensor. Keith Burns also gave two morning lectures that were background on nonuniformly hyperbolic dynamical systems and on how the general Hopf argument works to prove ergodicity in many such systems. These lectures touched on the recent result of Burns, Masur, Wilkinson that the Weil-Petersson geodesic flow is ergodic.

Yair Minsky gave a lecture on the Kahn Markovic theorem on how the mixing of the geodesic frame flow on a closed hyperbolic three manifold can be used to produce closed surfaces immersed in these manifolds. An ambitious and important conjecture raised at the conference is whether the Weil-Petersson metric can similarly be used to produce closed-surface subgroups of the mapping class group consisting entirely of pseudo-Anosov elements.

Dick Canary gave a background lecture on the theory of representations of surface groups into higher rank Lie groups, along the way introducing the Hitchin component of representations and the theory of Anosov representations. This was followed by a background lecture by Mark Pollicott on the thermodynamic formalism, the pressure metric, and how the work of Thurston, Wolpert and Bonahon, allows one to recover the classical Weil-Petersson metric from the thermodynamic formalism. This was followed by a lecture by Andre Sambarino on his joint work with Martin Bridgeman, Richard Canary, and Francois Labourie on how one can find a pressure metric on spaces of representations into higher rank Lie groups.

The lectures on the last day were by Ursula Hamenstadt and Jeff Brock. Hamenstadt presented her work on conjugacies between the Teichmuller and Weil-Petersson flows and Brock lectured on the relationship between the pants graph of a surface, the Weil-Petersson metric, and various objects associated to a Weil-Petersson geodesic, such as the ending lamination.

Tuesday afternoon was devoted to proposing a wide variety of problems and then the last three afternoons were spent in smaller discussion groups. One group considered the problem of counting closed geodesics in moduli space and large-genus limits of the geometry of the moduli space. The counting problem is delicate because the number of closed geodesics less than a given length is infinite due to the incompleteness of the metric and Brock "twisting". Various ways of resolving this issue were discussed. A second group discussed issues arising from higher rank representations. By the end of the week this group considered what classical properties of the Weil-Petersson metric might be proved directly from the thermodynamics formalism and seemed to have a proof of the incompleteness. A third group discussed how geometric properties of surfaces might be related to the Lyapunov exponents of the Weil-Petersson flow. Most of the problems considered in the afternoon sessions were in their infancy. The organizers felt there were healthy discussions with interesting leads and important feedback for the participants to continue their research. The workshop also provided opportunities to share expertise across an interested audience.