Relative trace formula and periods of automorphic forms

organized by
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Workshop Summary

Below is a list of the main themes of discussion in the conference.

One theme was a conjectural description of a formalism relating period integrals to the principle of functoriality and automorphic L-functions. Part of this effort, was the description of a general framework for the various relative trace formulas discovered so far. In particular, one of the goals was to arrive at a precise formulation of a general Gross-Prasad-Ichino-Ikeda type conjecture relating the square of the period integrals to special values of L-functions and local harmonic analysis. This is related to recent work of Sakellaridis-Venkatesh. A crucial remark was made for the seemingly trivial case where the period subgroup is the diagonal in a product. By considering L-packets, it became possible to fit this case in the general framework. There was also some discussion about the non-tempered case. Gross-Prasad Ichino-Ikeda formulations are not available when there is no local uniqueness. The lecture of Omer Offen shed light on the case of the unitary periods.

A related theme was the conjecture of Gross-Prasad for unitary groups and generalizations of the Gross-Zagier formula for unitary groups. Jacquet and Rallis have proposed a relative trace formula approach to the Gross-Prasad conjecture. The relevant fundamental lemma has been proved recently in positive characteristic by Zhiwei Yun. At the conference it was established that methods of logic can be used to extend this result to the case of characteristic 0 (and large residual characteristic). On the other hand, Shu-Wu Zhang has proposed a relative trace formula approach to the Gross-Zagier formula with possible generalizations to unitary groups in n variables. Again, a crucial step is a new kind of fundamental lemma where derivatives of orbital integrals appear. Preliminary work on the case of $n = 3$ was presented at the conference.

A third theme was the relation between the relative trace formula and the Weil representation. Zhengyu Mao gave a definitive exposition of the subject.

A fourth theme was the local theory. Fiona Murnaghan presented a lecture on (local) distinguished representations. In particular, she recalled the concept of a distinguished representation which is relatively cuspidal or relatively discrete series. This suggested revisiting the case of the symplectic group as a period subgroup. This also led to a question: what is the correct notion of an distinguished automorphic representation which is relatively cuspidal? Separately there was a preliminary discussion of the notion of a local relative trace formula.

Finally, there was a discussion of the relation with analytic number theory. This included the possibility of using the relative trace formula for obtaining estimates on L-functions. Some results in this direction were previously obtained by Ramakrishnan-Rogawski and Feigon-Whitehouse.