

# THOMPSON'S GROUP AT 40 YEARS

organized by  
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## Workshop Summary

Great scientific progress was made during the Thompsons group at 40 years conference on delineating the relationship of this remarkable group to diverse areas of mathematics, listing the major open questions connected to this group, and codifying approaches to those problems.

One unique aspect of this conference was the inclusion of Richard Thompson, after whom the group was named. Mr. Thompson has been an elusive figure in the mathematical world, and this workshop provided the opportunity to meet him, and hear his thoughts on the group. It was an educational as well as a historical opportunity. Several of the participants were quite involved with clarifying the history of the group, and documenting the motivations of early researchers connected to this group. Perhaps we should encourage one of them to write a historical manuscript documenting this information. The photographic record of the conference provided by Matt Brin, focusing in particular on Thompson but capturing all participants, was also quite unusual. Our workshop had three great strengths:

- the series of lectures

- the problem sessions, and

- the amount of interaction and collaboration which occurred during the meeting itself.

The lectures, usually two per day, gave introductions to the many different methods of approaching Thompsons group F. They always generated conversation that lasted far beyond the lecture time, mostly from eminent researchers who had never considered the group from a particular point of view before. We managed to expose all participants to these different perspectives without overwhelming them with lectures.

The second strength of the conference was the series of problem sessions. These generated such outpourings of ideas that it was hard to keep track of them all. The first problem session, on the question of amenability for Thompsons group, generated at least ten possible variations on the amenability question and approaches which might be feasible. The wealth of knowledge that was shared and debated was truly phenomenal. It was most certainly interaction between groups of people who would never have come together in this type of setting, all exchanging ideas about the same question.

Our second problem session was less focused, and not as vibrant. However, ten general problems were proposed and debated, with participants often disclosing partial results they had obtained in those directions. Presenting a problem was like reading an interactive book; instead of searching the index for a reference or theorem, there was always someone in the room who had either proved the theorem that was needed, or knew a good reference. This style of interaction will save many participants hours, if not days or months, of work.

Many of these problem sessions also led to discussions of smaller problems which might be appropriate for students entering the field.

Our third problem session was the most dynamic of all, and the longest, and dealt with fourteen areas of mathematics in which Thompsons group played a role. Group theory was not on the list, since the group theoretic questions relating to Thompsons group had been thoroughly explored in previous problem sessions. This problem session was at the heart of why we proposed this conference; we had the goal of broadening the horizons of the mathematicians working on this group and facilitating interactions exactly like this. The final problem session allowed for questions which had not fallen under the purview of the previous sessions. Nineteen different topics were raised, once again underscoring the diversity of this group.

During the workshop, there were small groups of people speaking in every corner of the workshop space. For every question one might have, an expert was within feet. This was quite a remarkable gathering of people to have in one room. We can only comment on personal collaborations which arose out of this conference. There was the opportunity for Taback and Cleary to renew collaboration with Burillo, and Stein, leading towards many planned meetings. Stein has obtained a new result recently about one of the generalizations of Thompsons group. Stein will visit Taback in Maine this spring, and Taback and Stein are going to Spain in March to work with Burillo. Cleary will visit Burillo in April, and has already visited Cannon and Woodruff in Utah to discuss material from the conference. Taback is working on a question discussed with deHornoy at the conference, and has been in touch with him recently. Cleary has been invited by Sergiescu to speak in Grenoble, France, and will visit Hermiller in Nebraska in March. There had been an increase in communication between Cleary/Taback and Guba/Sapir, who often work on similar metric properties of Thompsons group.

#### ORGANIZER COMMENTS

1. It would have been a great idea to publish the titles of the problem sessions. We did not do this mainly because we were not sure that we would have one each day. After the first day, it was clear that we would, and listing the topics would have been helpful to the participants, and saved us from answering many questions.

2. It would be a good idea to send the list of initial invitees to an outside reviewer, perhaps just one of the invitees, to make sure that there are no glaring omissions.

3. It would be helpful to have a list of what needs to happen for the organizers. I know that AIM likes to have things done informally, but there were certain things that had to happen, like the evaluations on the last day. I am envisioning a list of, Day 1: meet with AIM organizer morning, and after lunch,... Day 4: Check in with AIM organizers at break, and do evaluations. Just so we know when you would like to officially check in with us, and when things like evaluations have to happen.

4. I think that some of our success was due to the fact that we encouraged people aggressively to submit their statements to the web page. I think that future organizers should be this aggressive as well! I recall that participants were asked to submit a statement detailing their interest in Thompsons group; I think this wording could be expanded to ask for a list of problems. Many people did this for our workshop, and those were exactly the people who showed up with even longer lists of problems.

5. Ross Geoghegan organized his problem session extremely well. His method was to initially list about 15 areas from which we would discuss problems, with participant input. This helped us to cover a lot of material, organize the session well, and it helped more people to participate. I think the moderators should be told more emphatically that the purpose is to list questions; both Ross and Jose were excellent about not letting the sessions become too exposition oriented. They moved quickly on when it was clear that the questions had been well stated and explained.