Tensor decompositions

organized by
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Workshop Summary

Goals of the workshop

The Workshop on Tensor Decompositions, sponsored by AIM and the NSF, was held at the ARCC on July 19–23, 2004. Though higher-order tensor (also known as multidimensional, multi-way, or n-way array) decompositions have been around for more than three decades, the door is now opening on greater mathematical understanding and new applications. Previously, this topic has been the domain of researchers in psychometrics and chemometrics. Now, however, computationally oriented mathematicians have begun to take an interest and envision many more potential applications ranging from image and signal processing to data mining and more. The challenge is to find ways to extend these methods to larger data sets, i.e., data sets with thousands to millions of entries. This will require advances in the theory and computation of higher-order tensor decompositions.

The Workshop Itself

It is our understanding that the number of applicants for this workshop was unprecedented at AIM, and we were able to select an ideal set of participants with a variety of backgrounds in scientific computing, linear algebra, and various application domains. The participants came from Europe, Russia, and America, and ranged from graduate students to professors emeritus. There was also a lot of local interest with several non-funded attendees coming from Stanford and other local institutions. We had a total of 38 participants.

During the five-day workshop, participants (many of whom were non-experts) learned about tensor decompositions through a variety of talks. In addition to the introductory talks, the speakers (both pre-arranged and impromptu) also explored:

(1) real-world applications of tensor decompositions in chemometrics and image processing
(2) mathematical applications of tensor decompositions in independent component analysis and preconditioning,
(3) mathematical properties of tensor decompositions including how many entries of the tensor can be "zeroed out" and properties of the decomposition manifolds
(4) possible future applications of tensor decompositions in domains such as genomic signal processing and random sampling

Much of the group and break-out discussion time was dedicated to figuring out how tensor decompositions relate to well-understood matrix decompositions such as the singular value decomposition.

Quotes from participants

Here we provide a sampling of quotes from the participants taken from an informal survey eight months after the workshop:

I have made several contacts to people in new areas, with which I hope and plan to collaborate. Some of the things I think are going to be coming areas for collaboration are: improvement of algorithms, better knowledge of the peculiar properties of multi-way arrays and new application areas. I have made at least five new contacts (which would not otherwise have been made), some of which I already have regular communications with and others which I will later. I am quite sure that in 10 years we can look back at a lot of significant results which can all be related to the collaborations starting at the workshop. - Rasmus Bro, KVL, Denmark

The workshop significantly contributed to putting tensor decompositions on the table as new and important tools for numerical analysis, signal processing, data analysis,... The workshop was a great opportunity for meeting colleagues working on or interested in tensor decompositions. - Lieven De Lathauwer, ETIS, France

As a result of the workshop we have a clearer picture of the open problems. The connections made between tensor rank and certain generalized eigenvalue problems was particularly illuminating. - Charles Van Loan, Cornell University, Ithaca, New York

The workshop gave a deeper insight than I had earlier from the literature. The discussions on Matlab implementation and the Matlab tensor class (Bader, Kolda) helped to create mental images of operations on tensors, by making it possible to easily manipulate tensors. - Lars Eldén, Linkoping, Sweden

This workshop has been extremely useful to me. First of all, I learned a lot from Richard Harshman and Rasmus Bro about Tucker models and Parafac. I also collected several useful references, for instance on typical rank, among others. Next, I have been very happy to meet Lek-Heng Lim [Stanford graduate student], with whom I have started some very useful discussions, in particular on topological issues of tensor spaces. - Pierre Comon, University of Nice, France

I was greatly inspired and excited by this event. I went with some skepticism but was much enlightened by the great lectures and discussions. - V. Pereyra, Weidlinger Associates, Los Altos, CA

Sampling of outcomes

C. Martin (graduate student) and M. Kilmer wrote a summary of the workshop that appeared in SIAM News. url - http://www.siam.org/siamnews/11-04/tensors.pdf

P. Comon and L. De Lathauwer are organizing a follow-on workshop: Workshop on Tensor Decompositions and Applications, CIRM, Luminy, Marseille, France, Aug 29 – Sept 2, 2005. url - http://www.etis.ensea.fr/∼wtda/

B. Bader (postdoc) and T. Kolda have released software that consists of a set of MATLAB classes for manipulating tensors and their decompositions. They have an accompanying paper submitted for publication in ACM Trans. Math. Software. They are currently working on extending their software to handle the special case of large-scale sparse tensors.

R. Harshman has used the aforementioned software to develop a new data analysis method which has resulted in an abstract submission to the Psychometric Society.

L.-H. Lim (graduate student) reports several new connections, has three new manuscripts, and has written a new software package in MATLAB for computing a CANDECOMP/PARAFAC model.

L. Riechel is supervising several student projects on tensor decompositions.

L. Eldèn and R. Bro have organized a session on tensor methods at the upcoming ERCIM workshop in Copenhagen. url - http://www.models.kvl.dk/ercim/

Numerous participants have written and submitted new manuscripts as a result of the workshop, including P. Drineas and M. Mahoney, V. Fernando, M. Huhtanen, and others.