Subconvexity bounds for L-functions

organized by
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Workshop Summary

The primary aims of the workshop were:

(1) Understand and consolidate present methods of proving subconvexity.
(2) Begin analysis of higher rank subconvexity problems.

and of course an important side goal was:

(3) Encourage collaborations (old and new) among workers in this subject.

It seems that the workshop went a considerable way towards facilitating these goals, most particularly **(1)** and **(3)**. **(2)** was being worked on independently and the workshop did allow some interaction between the groups working on it, but it does not seem like any significant progress has yet been reported as a result of the workshop.

Concerning **(1)**: While we still have, at present, no explicit "unified theory" of subconvexity on $GL(\mathbf{2})$, the workshop went a good way in this direction. One striking example: it became clear during the workshop that the theme of "identities between families of L-functions" was one that was (independently) being used in the work of Jutila-Motohashi, Michel-Venkatesh and Bernstein-Reznikov! Moreover, it seems that this work is not "triplication" but genuinely different facets of the same principle. Even this observation alone seems like a satisfactory outcome for goal **(1)**.

Concerning **(2)**: Several groups of people have begun to work and discuss the basic higher rank families, particularly $GL_3 \times GL_2$. We note in particular Garrett (who talked on his work with Diaconu/Goldfeld) and Silberman. A "classical" version of this problem (accessible by the more-standard techniques of analytic number theory on $GL(\mathbf{2})$ esp. Kuznetsov) was proposed by Venkatesh. We hope that the different groups studying this problem will continue to interact.

Concerning **(3)**: This seems to have been by and large successful, although see notes below on possible improvements to the workshop format. Several of the established groups who work on the subconvexity topic (Liu/ Yangbo/Ye; Bernstein-Reznikov/ Michel-Venkatesh/ Harcos) had ample time for continuing their collaboration as well as interacting with each other. Moreover, some new working groups were formed through AIM's policy of encouraging colloborative work during afternoons. In particular, the group of Lillian Pierce appeared to make an interesting start on obtaining strong subconvex bounds for Dirichlet characters with highly composite modulus and the group of Matt Young made some progress on deriving identities between families of L-functions via geometric methods.

Possible changes/improvements in retrospect:

It seems that a serious mistake made by the organizers was not suggesting sufficiently precise problems to the working groups on "Periods". While the members of those groups appeared to enjoy and benefit from their experience (at least anecdote suggests as much), nevertheless, the problems posed were too diffuse; as a result of which the group fragmented into many parts. In general it seems like choosing focussed problems for the groups at the start is VERY important.

Concerning the workshop format, the following comments are made by Venkatesh and were seconded by Duke. The idea of forcing people to work in groups works really rather well for those who are not already working in the area and for graduate students. (This is a comment that has been seconded by various others I have talked to about AIM workshops). These people would be left out without such an active process. On the other hand, it can be VERY disconcerting for those of uswho already have worked on the topic. I found it often unpleasant to be forced myself into working groups, and to be in the position of forcing others into working groups.

So I would strongly suggest some kind of balance, whereby people who wish to work in a focused group of their own may do so, so long as they are still willing to give brief reports on their progress and on the questions they are considering. Admittedly I do not think it was the intent of the AIM policy to disrupt existing collaborations in this way, but nonetheless, that was the effect to a certain extent.