Moduli spaces of knots

organized by
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Workshop Summary

Narrative

Our workshop was the first conference in the field of moduli spaces of knots, bringing together people who have studied these spaces from four distinct approaches, as well as researchers from closely related fields. One thing we immediately found was a strong tie with the study of knot energy functionals. Thankfully the participant list had plenty of people from this field who had applied. The main accomplishments of the workshop involved cross-pollination between these subfields of: three-manifold techniques, calculus of embeddings, knot energy functionals, finite-type knot invariants, singularity theory, and quantum field theory. No big results were proved during the conference, but the interactions - often informal (around sketchpads set up in the back of the room) - should deepen the work done in all of these areas.

We can list some of the work which developed as a result of the conference, only as we organizers are aware. Conant submitted a paper which relates the embedding calculus homotopy spectral sequence to primitive weight systems. Conant and Sinha have started a project on the finite-type group structure on the invariants coming from this spectral sequence. Turchin and Sinha have started a project relating this homotopy spectral sequence to the homology spectral sequence. Budney improved his control of the structure of the space of prime long knots in $\mathbb{R}^3$. Budney and Cohen used this information to determine the least degree for which p-torsion, p odd, occurs in the homology for the space of long knots in $\mathbb{R}^3$. The interesting feature here is that such p-torsion arises for the same reasons (as well as dimensions) as the first occurrence of p-torsion in the stable homotopy groups of spheres.

Brief summary of activities

Day One Survey talks in each of the four main approaches to knot spaces:

- Volic showed how the calculus of embeddings gives clean models when applied to spaces of knots.
- Turchin surveyed Vassiliev's original approach to these spaces through singularity theory, as well as some of the combinatorics which arises.
- Budney gave the fairly complete picture in dimension three, where three-manifold techniques combine well with algebraic topology.
- Longoni defined integrals from quantum field theory which explicitly give cohomology classes in Euclidean knot spaces.

In the afternoon problem session led by Goodwillie, we started with some of the basic questions in the field and then moved on to questions which connect this sub-field with others. There was a strong demand for survey talks in some areas, including knot energy

functionals, which had not been on the original agenda. Despite all of the amazing recent progress, there was a desire to work together on some basic questions on the homology of these spaces and their relation to knot theory. Some audience members shared knowledge of unpublished examples which were shared later.

Day Two The morning started with talks in geometric knot theory. John Sullivan and Jun O'Hara gave a thorough survey of the state of knowledge about knot functionals, complete with figures and animations. Jorge Calvo did the same for "stick knots."

In the afternoon there were talks on the calculus of embeddings. Goodwillie explained his original motivation for the theory, starting with the Hirsch-Smale theorem and organized by higher-order Mayer-Vietoris theorems. Sinha then gave more details about how this general theory leads to a concrete geometric model in the case of knots.

Day Three There were two main groups. One focussed on the homology of knot spaces, talking among other things about the relationship between what is known in dimension three and in higher dimensions. Another group focussed on knot invariants through evaluation maps, trying to generalize the "quadrisecant result" and to better understand its significance in connecting with geometric knot theory.

This group work lasted most of the day, but before and after lunch there was a talk by Lambrechts on the collapse result for the rational homology spectral sequence for the space of knots. This collapse result resolves a famous conjecture by Kontsevich made at the European Congress of Mathematicians in 1994. Lambrechts, Turchin and Volic took the opportunity to collaborate on this work, still being written, at various points in the conference. There was also a short talk by Longoni giving the conjectured form of a cycle in the space of knots.

Day Four One group worked on bounded embeddings as a way to deloop these knot spaces (which at least monoids and often two-fold loop spaces). They determined that the calculus of embeddings is not suited in its current form to understand bounded embeddings. Another group continued on knot invariants through evaluation maps. In the middle of the day Hatcher spoke on the foundational results which allow one to say so much in dimension three. In the afternoon a group talked about new torsion weight systems in finite-type invariant theory, starting with a talk by Stanford on some unpublished work of his.

Day Five There were no structured activities. Many of the small groups which had ongoing projects met: Conant-Sinha-Longoni on finite-type invariant structures; Lambrechts-Volic-Turchin on their collapse result; Brendle-Denne-Sullivan on geometric knot theory; Munson-Chernov on higher-dimensional questions.