Selfsimilar groups and conformal dynamics

organized by
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Workshop Summary

The principal goal of the workshop was to bring together researchers in a number of very different areas (conformal dynamics, group theory, number theory, operator theory, Gromov hyperbolic spaces) so that newly discovered connections between the topics of self-similar groups and conformal dynamics could be explained, developed further, and applied to problems in a timely fashion.

Progress. The workshop seems to have achieved its primary goal. Experts in conformal dynamics learned the new algebraic techniques developed by leading specialists in asymptotic group theory such as V. Nekrashevych, M. Sapir, M. Abert, and Z. Šunić. Experts in group theory learned of the continuing source of interesting examples furnished by conformal dynamical systems and multidimensional rational mappings (like Hénon maps). There were also other groups of mathematicians working in C^*-algebras, topology, and probability theory. It was noteworthy that each group was able to understand the other mathematicians.

Developments. Areas of special resonance included the following:

- The relationship between C^*-algebras and dynamical systems intrigued many participants. Kaminker pointed out that it is known that a nice relationship exists between the algebras associated to doubling on the solenoid and to the action of the modular group on its limit set. Nekrashevych mentioned a close relationship between the Ruelle algebras and those of self-similar contracting groups. It is clear now that computation of the K-theory of algebras related to self-similar groups is an important problem.
- Nekrashevych's assignment of a virtual endomorphism of the mapping class group to a homotopy class of postcritically finite branched covering explains the detailed structure of an example studied by Cannon, Floyd, and Parry.
- Nekrashevych observed that if one could verify that IMGs of so-called dynamical Belyi polynomials were weakly branch, then this would establish that these IMGs are not Galois invariants of the action of the absolute Galois group, answering a question of Pilgrim.
- The functoriality of Nekrashevych's constructions give an avenue for attacking a tough problem in holomorphic dynamics: find a postcritically finite rational map with Julia set equal to the whole sphere which is provably not obtained via the operation of "mating".
- Vorobets proved the existence of a selfsimilar free group; Perez came to Vorobets' lecture, and then discussed a new proof of this result.

- It became clear that a result of M. Abert and B. Virag implies that non-elementary hyperbolic groups (more generally, hereditary sub-directly irreducible groups, which are not virtually nilpotent) can not be contracting self-similar. This and other observations due to Abert and Steinberg give an approach for proving the interesing conjecture that contracting self-similar groups do not contain free noncommutative subgroups.
- I.Pak described how self-similar groups could be useful in study of percolation on groups and graphs
- It became clear from the talk of Smillie that there exists an interesting relationship between contracting self-similar inverse semigroups and Hénon maps.

New collaborations. It seems likely that the following collaborations during the workshop will lead to joint projects:

- Nekrashevych-Pilgrim discussed giving an algebraic formulation of Thurston's criteria for a postcritically finite branched covering to be equivalent to a rational map. A sketch of a proof was given.
- Kaminker-Nekrashevych have agreed to collaborate more on operator algebras associated to expanding dynamical systems. Tools of Nekrashevych provide a symbolic description of Ruelle algebras of "generalized solenoids" and the theory developed by I. Putnam, J. Kaminker and others gives an understanding of the duality between the iterated monodromy groups and their limit spaces. But further clarifications are necessary.
- Grigorchuk and Sapir started to think about the growth of Dehn functions of self-similar groups of intermediate growth.
- Grigorchuk and Kharlampovich focused on algorithmic problems related to self-similar groups such as the problem on solvability of equations and the Tarski type problem on the decidability of the elementary theory in self-similar groups.
- Kaimanovich, Nekrashevych, Epstein, Virag and Grigorchuk discussed various topics related to amenability of self-similar groups and to applications of amenability in conformal dynamics and in probability.
- Sunik and Hermiller discussed topics on geometric properties of self-similar groups, as well as problems on torsion growth.
- Smillie and Vorobets discussed topics related to billiards and to substitutional dynamical systems.

Prospects for the future. The injection of new algebraic techniques into conformal dynamics will surely change the level of interest in the field to the broader mathematical community. The implications of amenability for IMGs of postcritically finite polynomials continue to be mysterious and will likely receive attention.

Great perspectives come from the possibility of using the methods and ideas of conformal dynamics in group theory, in particular toward solving the isomorphism problem.

Abert has demonstrated the power of probabilistic methods in study of group actions on rooted trees. The development of these methods looks very promising.

A number of interesting projects came from Kaminker's lecture and the discussion session led by him. This should shed light on the possibility of using self-similar groups and

operator algebras generated by them to the computation of Hausdorff dimension (and other types of dimensions) of fractal sets. This relates well with the ideas of Zoran Šunić presented by him in discussions around the "Hanoi Tower" groups and other types of self-similar groups.

A number of interesting ideas were expressed by participants regarding the solution of some algorithmic problems for self-similar groups. The methods of dynamical systems may be used to solve some problems of this type, in particular the isomorphism problem and the problem of estimation of growth of the Dehn function.

Very interesting ideas were expressed by J. Cannon on exploration of the self-similarity phenomena related to hyperbolic groups.

Synergistic activities. Apart from four formal lectures scheduled in advance, there were a number of discussion, problem, and hands-on working sessions:

- Two problem sessions were run: one on selfsimilar groups and one on algorithmic problems for selfsimilar groups.
- Following a request by a participant, a "recitation" session led by Sunik was held after his formal lecture, where the calculus of computations in selfsimilar groups was explained in concrete examples.
- A number of younger researchers and a few older non-experts attended a "working" session led by the expert Nekrashevych in the computation of the recursion for an explicit example.
- Participants contacted Grigorchuk and Pilgrim as organizers with requests for specific topics by speakers. Such lectures were delivered by Kaimanovich on amenability, by Cannon and Pilgrim on finiteness principles, by Sapir on residual finiteness, and by Smillie on Henon maps.
- A universally liked feature was that typically, there was a minimum of an hour between formally scheduled activities (this interval was shortened when there were parallel activities).