RY AND REPRESENTATION THEORY OF TENSORS FOR COMPUTER SCIENCE, STATISTICS AND OTHI

organized by
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Workshop Summary

Introduction. This workshop was sponsored by AIM and the NSF and it brought in participants from the US, Canada, Japan, and the European community to Palo Alto, CA to work to translate questions from quantum computing, complexity theory, statistical learning theory, signal processing, and data analysis to problems in geometry and representation theory. This was the first coordinated workshop between AIM and MSRI, as there was a two-week Summer Graduate Workshop of the same title immediately preceding it. The structure worked well. The organizers as well as AIM participants Bernardi, Boralevi, Comon, de Silva, Friedland, Gross, Kondor, Nishimori, Oeding, Ottaviani, Qi, Yang, and Ye joined for some or all of the MSRI workshop, with several giving talks. The preceding MSRI summer graduate program was essential for giving us a head start in working out a common language and identifying problems of common interest. P. Comon was especially effective at translating engineering problems to geometry, as reflected in the remarkable success of the signal processing group at AIM.

In each of the areas of applications, varieties in spaces of tensors invariant under linear changes of coordinates appear as central objects of study. Despite their different origins, there are striking similarities among the relevant varieties and this workshop focused on exploiting geometric structure to produce new results in the fields of application. Five introductory talks were given by Cai, Comon, de Lathauwer, Gour, and Valiant, with short talks by Bürgisser, Gross, Landsberg, Morton, and others to elaborate on topics that arose in dicusssions.

On the first afternoon, we split into two groups: signal processing and computational complexity. On the second day, we broke into four groups which persisted for the duration of the workshop, with some participants participating in several different groups:

(1) Multi-linear techniques in data analysis and signal processing,
(2) Geometric approaches to P?=NP,
(3) Matchgates and holographic algorithms, and
(4) Entanglement in quantum information theory.

Each group contained a mix of practitioners and geometers. The groups first went to work on translating problems into a language which could be understood by both parties. In some cases this took the bulk of the workshop; in others work quickly progressed to working out examples, applying known results to new areas, and stating, and even making progress on, open problems. Some of the results of this work are described in the draft technical report compiled by L. Oeding which is attached.

Quantum information theory. Some progress was made during the workshop on the additivity conjecture in quantum information theory; for example, they determined some necessary conditions for the conjecture to hold. They also formulated a local version of the problem. The counter-example to multiplicativity for tensors (which is related to the additivity conjecture in QIT) presented by Gross was re-phrased in elementary geometric language by Derksen and Manivel.

Mulmuley-Sohoni approach to $P? = NP$. A major goal of this group was a clear translation of the Mulmuley-Sohoni approach to the language of Geometric Invariant Theory. The group highlighted the role of the Kronecker coefficients in calculating coordinate rings of the GL_{n^2} orbits of the determinant and permanent polynomials. Another project was describing Weyl group invariants for the action on zero weight space of an irreducible representation (some incremental progress was made in actual calculation) and explaining the role of this problem in relation to the coordinate ring of the GL_{n^2} orbit of the permanent polynomials. This helped clarify the main difficulty: relating the coordinate ring of an orbit and the coordinate ring of its closure. The group also brought out the role of subspace varieties as we go from the orbit of the permanent in m variables to the orbit of the permanent times a multiple of a linear form, and again from that to the GL_{n^2} orbit of such (where $m > n$).

Signal Processing. The signal processing group benefited from phrasing some problems they care about in the language of algebraic geometry. The result was an awareness that there is significant information regarding many of their open questions already available. Examples include the relationship between weak defectivity and uniqueness of tensor decompositions, and and making connections to the algebraic geometry literature on subspace varieties which were referred to in terms of multilinear rank by engineers. In other cases new results were obtained during the workshop; for example, progress was made on whether symmetric rank and tensor rank are equal for a symmetric tensors by Gross (who works in quantum information theory!) and others. As tensor decomposition is an important theme, the understanding of cases where there is a finite or tame orbit structure (and so a meaningful noiseless decomposition may be available) as studied by Kac and others was also of interest.

Matchgates. The group with the slowest start-up was the Matchgates group, due to a combination of two factors; first, the workshop participants aside from Valiant, Cai, and Lu had little previous exposure to the topic, and second, none of the three experts had been able to join for the MSRI session to provide a head start. To remedy this two of the organizers (Landsberg and Morton) devoted the majority of their small group time to developing a common understanding among the theoretical computer scientists and geometers. We made sufficient progress that we are able to continue working towards stating the essential open questions regarding matchgates and holographic algorithms in geometric language and to eventually to work on these questions. One immediate consequence was the recognition of the Spinor varieties in the space of possible matchgates.

Conclusions. The AIM format was new to many of the participants, but we adapted quickly. Once the four groups had formed, the critical 4-7 person size was reached and real work started getting done. Many conversations overflowed the bounds of the workshop into

the evening. Drifting among the groups for updates, the organizers heard mentions of what would go in "the paper," and thus we are hopeful that multiple publications will result from the collaborations sparked here. Further we can be certain that some of the development of common language and expositions of what is known and its relevance will appear in the forthcoming Landsberg and Morton book.