Random analytic functions

organized by
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Workshop Summary

The organizers and other participants gave introductory talks to introduce basic concepts in their fields and to propose directions for research. A. Dembo and O. Zeitouni introduced some basic probabilistic methods: Slepian inequalities, large deviations, Kac-Rice formulas, and concentration of measure. M. Sodin gave an introductory talk on random analytic functions of one complex variable, particularly on Offord type estimates for hole probabilities. S. Zelditch gave an introductory talk emphasizing geometric aspects of Gaussian random polynomials and their asymptotics as the degree tends to infinity. B. Shiffman described some more advanced results on the geometry of critical points of random holomorphic sections, and their connections with Sodin-Tsirelson's new analysis of random gradient vector fields in the plane. J. M. Rojas described new quantitative estimates on the topology of real fewnomials zero sets. Y. Peres and B. Virag gave a talk on their joint paper on their determinantal zero point process. S. Sheffield gave an exposition of the Gaussian free field, concentrating on his joint work with O. Schramm on defining the random zero set of the GFF by using a discrete approximation.

Each of these topics was developed in further detail in discussion sessions. In a session on hole probabilities, M. Sodin explained the methods of his asymptotic result with B. Tsirelson on hole probabilities for zeros of entire functions in disks of increasing radii. Discussion revolved around which parts of the proof depended on the special geometry involved and how it could be generalized. S. Zriebec gave his generalization of the Sodin-Tsirelson result to higher dimensions, and some participants provided heuristic reasons for this estimate.

A. Guionnet and S. Zelditch led a discussion of large N limit problems, contrasting the large deviations of empirical measures of eigenvalues of random matrices and of zeros of random polynomials. S. Zelditch presented results obtained during the workshop of the rate functional for empirical measures of zero point processes of special polynomial ensembles, which raised a discussion on its relation to work of Guionnet-Ben Arous and Zeitouni-Ben Arous on random eigenvalues, and also on its consistency with results presented by T. Bloom on equilibrium measure of zeros.

J. M. Rojas led a discussion on a refinement of Khovanski's Theorem on Complex Fewnomials. In particular, Rojas' conjectural refinement involves discriminants and polyhedral combinatorics, and the univariate case was a focus because of its apparent tractability. D. Farmer and S. Zriebec made valuable suggestions.

O. Schramm and S. Sheffield led a discussion of the Gaussian free field and its relation to the SLE equation. Their discretization of the GFF was contrasted with the polynomial approximations used in other talks. Sheffield also discussed flows of certain random vector fields in the plane. M. R. Douglas posed a number of problems on Gaussian random gradient vector fields and their flow times which have come up in astrophysics and cosmology.

Other highlights included two problem sessions and a panel discussion, transcribed by S. Sethuraman. Among many interesting problems, a similarity between random tilings and Viro diagrams was observed, and a discussion was held on which references would be valuable for graduate students who want to enter this area.