

STOCHASTIC AND DETERMINISTIC SPATIAL MODELING IN POPULATION DYNAMICS

organized by
Zhilan Feng and Priscilla Greenwood

Workshop Summary

The mood of the workshop was one of great interest and even euphoria. Several people said it was the best workshop ever, and greatly preferred to the usual meeting format. It was commented that our choice of invitees was excellent. This was a genuinely new gathering of people who had not been acquainted before and were very pleased to make connections, even at the senior level. We had good communication between stochastic and deterministic modelers, and the people already combining these approaches were inspiring to those not yet doing so. Of course the eventual output is yet to be seen.

We will tell about how the workshop went with emphasis on what we thought were significant points.

On Monday morning Alan Hastings talked on modeling spread of spartina grass, which grows in complex patterns on beaches, saying that stochastic modeling of this natural phenomenon was difficult. Fred Adler's talk suggested the problem of defining genetic space with something like a metric among DNA patterns. He pointed out that heterogeneity has been daunting, people are afraid to try to model it. On Tuesday morning Steve Krone talked about interacting particle systems (IPS) and reaction-diffusion equations being their limits by local stirring. Nicolas followed this up Wed with a lot more about IPS models which he said are all based on combinations of voter and invasion models, also IPS on trees, small world graphs, hypergraphs, dynamic graphs. Going back to Tuesday, Linda Allen talked about stochastic epidemic modeling and raised the question of what is R_0 (the critical parameter in many epidemic models) for spatial and for stochastic models.

The first afternoon we quickly listed several problem areas, formed groups of 4-6, and started working. Three of the groups started on Monday intend to continue working:

Alan, Judy, Natali: harvesting management with a spatial model

Fred, Zhilan, Sivan: defining a genetic mutation space

Frithjof, Cindy, others: effect of stochasticity on wave speeds using integro-differential equations with various kernels and compared with Fisher's equation

Wed morning Sebastian talked on a multiplicative noise model for environmental noise. Space is not explicit in this kind of model. A question arises about explicit vs explicit (e.g. IPS) spatial modeling.

On Tuesday afternoon a new group was formed on Allee effect (tendency of a small spatial population not to find mates and thus reproduce, clearly a topic relevant to spatial modeling, but usually treated with implicit space) and how stochasticity will effect Frank's Allee effect model. This group worked for the next 3 days with the problem coming into focus, and will continue: Frank, Linda, Li, Bassady, Cindy, Sivan.

On Wednesday afternoon we decided to form groups on different approaches to spatial coexistence, multi-species models models which predict coexistence in certain parameter ranges. Four groups formed:

- (1) ODE coexistence (comparing parameter ranges of coexistence for ODE and a related stochastic model): Zhilan, Zhipeng, Joaquin, Rongsong, Aziz.
- (2) coexistence (comparing times to extinction from a quasi-stationary coexistent state): Anuj, Sebastian, Carl, Cindy, Alan.

These two approaches are based on many patches which may change character from entirely species 1 to entirely species 2 or coexisting 1 and 2. There is a question (which we will clarify) how these models differ from the corresponding single patch models.

- (3) IPS results leading to coexistence applications: Nicolas, Sebastian.
- (4) Point process model for coexistence using Ripley's K function: Fred, Natali, Carrie, Frithjof.

A question arising from the last two approaches is: can the point process model be obtained as a limit from an IPS model for coexistence? Do the methods in IPS correspond to methods for a limiting point process model?

After the four groups worked, the four methods which had been formulated were written on the board in a long session which lasted late on Thursday. The next day these were put on the board again and comparisons made. The last 2 methods are specifically spacial and the first two are implicitly spatial. We intend to apply for an 8-person focussed research group on the comparison of these four approaches to coexistence at a new Tennessee research center, where Alan is involved, and will compose the request for this in August. These four groups will also continue to work on their approaches as separate groups.

On Thursday morning Frank talked on Allee effect and Frithjof on speed of propagation. These talks led to refocusing of two of the corresponding group topics already mentioned.

On Friday morning Rebecca compared 4 types of predator-prey models with application to the Canadian Lynx and Hare data. Zhilan reported on preliminary results of the ODE coexistence group which seem to be very promising.

Friday afternoon was devoted to wrap-up. All the participants who had not given presentations were invited to tell what impact the workshop had on them. Each one was extremely pleased and felt that he/she had made valuable contacts. They were amazed to see that

joint work can spring up rapidly in this kind of conducive environment. Several had become involved with specific research projects at the workshop.

Finally, we reviewed the topics of ongoing research which had been started and listed the members of the seven corresponding research groups, recorded above.

It was felt that the workshop had been a great experience for all participants, and that in addition to the initialized collaborations, our perspective on the topic had grown and sharpened. In particular our objective of encouraging research combining deterministic and stochastic methods appeared to be making gratifying headway.

We are greatly indebted to the AIM staff for their insightful guidance throughout the workshop!