Numerical Methods for Optimal Control in High Dimensions

organized by
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Workshop Summary

The goal of the workshop was to discuss techniques for solving high dimensional optimal control problems, the Hamilton-Jacobi-Bellman equation and/or dynamic programming, as well as applications which might benefit from these techniques.

The specific topics covered were shaped by the participants who were present. The subjects fell roughly into three areas:

(1) Solution of the Bellman equation by exact and approximate methods:
- Existence theorems for solutions of approximate dynamic programming problems in discrete time but (potentially) continuous space. The discussion included value and policy iteration, but focused on approaches based on (approximate) linear programming.
- Model reduction techniques for lowering the complexity of the discretization to be solved computationally.
- Semi-Lagrangian methods in an arbitrary number of dimensions (in practice up to dimension six).
- Ordered upwind methods for static, anisotropic Hamilton-Jacobi equations, with some time dependence.
- Numerical techniques included: finite difference methods, Markov chain methods, approximate dynamic programming, iterated control. Details of implementation and coding were covered in the discussions.

(2) Applications:
- Computer animation. We discussed driving musculo-skeletal models of birds to follow prescribed paths by nonlinear programming, extraction of model parameters for various human walking styles, and subtle modification of fluid flows to assume prescribed shapes computed via adjoint methods.
- Communication. We discussed state estimation and approximating and controlling a delay differential equation model of network flow control.
- Mathematical finance. We discussed analytic properties of the underlying second order PDEs, which are forms of the HJ PDE for stochastic systems. Finance was also mentioned as an application of approximate dynamic programming methods.
- Visibility problems from computer vision. We discussed how visible subspaces, paths with optimal visibility, and local perturbations of visibility can all be related to HJ-type PDEs.
- Collision avoidance and safety zones for aircraft control. HJ PDEs can be used to compute reachable sets, which arise in these examples of system verification.

  - Two biological applications were discussed. determining the optimal course of chemo and/or immuno therapy for cancer patients using a six dimensional model and adjoint based methods and a parameter estimation problem for discovering protein feedback pathways that govern fly wing development.

(3) State and parameter estimation arise frequently in optimal control. In isolation, the former gives rise to HJ-like PDEs, while the effect of the latter must be taken into account to produce robust practical solutions of optimization problems.
  - In communication, we discussed both particle and Kalman filter approaches to channel state estimation.
  - Biological applications where parameter estimation is a major challenge. Compared to many other engineering applications, there are no clear mathematical "first-principles" from which to build a model, and available data tends to be noisy, sparsely sampled and/or qualitative.

Most of the activity was based on informal discussions and breakout sessions on numerics and applications, which took advantage of the many expert participants.

Open problems that we identified include:

- Appropriate cost functions whose optimization generates "realistic" animation of animals and humans.
- Integration of exact and approximate dynamic programming methods into a single, tunable scheme.
- Choice of appropriate basis functions for approximate dynamic programming. This question is closely related to the general problem of avoiding an exponentially expensive discretization when working in high dimensional spaces (or even in six to ten dimensions).
- Improvement of conservation properties of numerical HJ solvers, which are key in application areas such as finance and fluid simulation.