Theory of motives, homotopy theory of varieties, and dessins d'enfants

organized by
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Workshop Summary

The meeting was held at the AIM offices in Palo Alto, April 23-26, 2004. The group of participants was very diverse in its intellectual focus. It included number theorists, workers on absolute Galois groups, and topologists who are studying motivic homotopy theory and the topology of algebraic varieties in more generality. The purpose of the meeting was to expose workers in each of these areas to techniques and questions in the other areas, with the idea of broadening the mathematical outlook of all the participants. The program included several series of invited lectures, a problem session led by Jack Morava, and several example sessions, in which participants demonstrated the use of techniques in their areas of expertise to work out specific examples. The philosophy behind these example sessions is that one often learns and appreciates techniques best when they are put to use in the case of a particular problem.

The expository lectures were as follows:

Leila Schneps (3 lectures) (1) Dessins d'enfant and Grothendieck-Teichmuller theory (2) Complexes of curves, moduli spaces, Thurston theory (3) Lie Grothendieck-Teichmuller theory and multizeta values

R. Jardine (2 lectures) (1) Stacks and homotopy theory (2) Motivic spaces and the motivic stable category

B. Kahn (2 lectures) (1) Pure motives (2) Motivic Galois groups

F. Pop (2 lectures) (1) Galois groups of function fields (2) Recovering fields from their Galois theory

M. Levine (2 lectures) (1) Mixed motives and cycle complexes (2) Mixed Tate motives

B. Toen (1 lecture) (1) Higher algebraic stacks and homotopy types of algebraic varieties

The example sessions included examples of how to work with and explicitly compute Dessins denfants (L. Pharamond, M. Wood), examples with the arithmetic fundamental group (J. Ellenberg), examples in motivic homotopy theory (D. Isaksen, D. Dugger), and questions about the motivic Galois group (B. Kahn).

The problem session generated interesting directions for future research.

(1) The clarification of both stable and unstable homotopy types of moduli spaces of curves will have strong consequences for many aspects of the general area covered by this meeting. In particular, the relationship between the compactified and uncompactified moduli spaces seems critical in understanding better arithmetic versions of recent work of Madsen and Weiss on the cohomology and homotopy type of these moduli spaces in the stable range. Understanding these questions is also important in carrying further the study of the Grothendieck-Teichmller group. A typical question to which one would like and answer

is whether or not there are actions of the absolute Galois group of the rational numbers or the Grothendieck-Teichmller group on the Madsen-Weiss construction.

(2) The motivic Galois group was perceived to be a very important object, which hasnt yet been studied with enough care, unlike its ordinary Galois theory counterpart. It was concluded that a more detailed computational understanding of this object is an extremely important objective.