

# $L^2$ HARMONIC FORMS IN GEOMETRY AND STRING THEORY

organized by

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## Workshop Summary

The goals of this workshop were to bring together a collection of mathematicians and physicists sharing an interest in problems related to metric asymptotics and  $L^2$  cohomology on instanton moduli spaces. An original motivation for this meeting was a set of conjectures by the physicist Ashoke Sen which use string-theoretic S-duality to predict dimensions of spaces of  $L^2$  harmonic forms on the the Atiyah-Hitchin  $k$ -monopole moduli spaces. The central issues here are arriving at a suitable understanding of the asymptotics of the natural metrics on these spaces, and then of understanding the dimension (and ideally the generators) for the space of  $L^2$  harmonic forms.

The participants included: a group of several mathematical physicists who are leaders in the recent developments in these  $S$ -duality predictions; several mathematicians who are experts on the geometry of monopole moduli spaces and many of the related techniques, e.g. Nahm transforms, or are experts on complete metrics of special holonomy on non-compact manifolds; finally, several mathematicians responsible for many of the significant breakthroughs in  $L^2$  cohomology in recent years.

The first three days of the workshop were devoted to introductory and survey lectures in each of these areas: Cherkis, Gomis, Cvetic and Etesi gave a series of interlinked talks concerning the different asymptotic geometries of instanton moduli spaces, the role of metrics of special holonomy in M-theory, the appearance of  $L^2$  harmonic forms as ‘forcing terms’ in these field theories, an overview of gauge field theory, and an attempt to provide some of the physical intuition behind  $S$ -duality; Singer and Jardim each gave lectures on the geometry of monopole moduli spaces and the Nahm transform; Saper, Carron and Melrose each gave talks on three distinct approaches to understanding  $L^2$  cohomology on complete manifolds.

There were open problem and moderated discussion sections after the first two days, and these were used to clarify a number of the open questions raised by the speakers in their talks. The fourth and fifth days were devoted to some more specialized talks: Saper described his recent work on L-modules and the  $L^2$  cohomology of locally symmetric spaces, Melrose described his approach to determining the  $L^2$  cohomology of singular algebraic varieties, and Cherkis gave the concluding talk, where he presented more details and some new perspectives on material he had talked about previously.

In addition, on Saturday afternoon, two postdocs, Yeganefar and Degeratu, gave short presentations of their own work, and Maartens, a graduate student, gave a general overview of equivariant cohomology and raised some interesting open problems. There was a final wrap-up session on the fifth day where a number of new open problems were raised and some of the main themes of the meeting were reviewed.

The consensus was that the meeting was very successful: the physicists did an outstanding job of explaining the context of their problems and translating them to a mathematical audience, and the various mathematicians gave a very good presentation of the leading analytic approaches which might be useful.

There was definite progress made and a number of potential collaborations were started: there is only a fairly loose physical intuition about the expected nature of the asymptotics of the metrics on the monopole moduli spaces, and this needs to be formalized, but it was made plausible during discussions at the meeting that most likely the correct tool to do this is the compactification theory developed by Mazzeo and Vasy in linear  $N$ -body theory; in other discussions, Carron realized how to generalize his basic notion of nonparabolicity at infinity to certain product manifolds, which gave him hope that there might well be a way to employ his approach, based on a refined Mayer-Vietoris principle and semi-local Poincare-type inequalities, to the monopole moduli spaces; Etesi and Carron had some very useful conversations on the possible use of Vaillant's index theorem for the infinitesimal approach to the Yang Mills instanton problem on Taub-NUT space, which might lead to some further collaboration; Cherkis, Etesi and Hausel have started to work on Yang-Mills instantons on Taub-NUT space; in discussions with the physicist Lee, Jardim learned of an open problem about the monopole moduli spaces which he thought he knew how to solve using his knowledge of the Nahm transform, and he went away confident that he would be able to do this; Hausel was excited to learn about DeGeratu's work on relating analytic (index-theoretic) data to the topology of crepant resolutions of finite group actions of  $C^3$ , and felt like there should be some relationship to his earlier work with Sturmfels and Proudfoot on the combinatorics of certain hyperKaehler quotients, and more generally felt that her work could have a serious impact on some of his ongoing work.

Altogether, enthusiasm was expressed all around, not only for the many stimulating discussions, but also for the unusual format suggested by AIM, with the more general talks and the moderated discussion sections. (Based on the success here, Bruening and Mazzeo plan to use this format in an Oberwohlfach meeting they are organizing next year.) While there was no specific plan to try to bring this particular group of people together again, there was no doubt that contacts made here would be kept up. Furthermore, many people departed feeling like they were exposed to some new techniques or ideas which would further their research goals. The problem sessions effectively outlined a number of important open questions, ranging from much longer range speculative ones to ones very immediately accessible.