The Klein project

organized by
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Workshop Summary

Purpose

The Klein Project is an IMU/ICMI initiative to provide the means for secondary teachers of senior classes to continue to engage their interest in mathematics, in particular to enable them to understand the breath, unity, and evolution of the contemporary research field. It draws inspiration from Felix Kleins work 100 years ago. The outputs of the Klein Project are to be a book that includes chapters and mathematical "vignettes", a website that can be kept up to date, and other resources.

Klein Project meetings have been held in six different countries so far, and have successfully used the space between school and university mathematics as a place where teachers and mathematicians can engage with each other. We wished to develop this interaction further.

The Workshop had several objectives:

- To develop a Workshop model that would enable teachers, educators and mathematicians to collaborate together in the production of written, computer, and web resources for teachers.
- To clarify the characteristics of Klein Vignettes.
- To identify, design, produce, and critique some Klein Vignettes.
- To progress the Klein Project into an intensive writing phase, planning a mechanism for continued Vignette preparation, authoring of chapters, and the print and web "life" of the project.
- To have a meeting of the Klein Project Design Team to set out a timetable and deadlines for future work.

The Workshop

Thirty participants, from Australia (1), Brazil (1), Canada (2), France (1), Germany (1), Italy (1), New Zealand (1), Spain (2), UK (1), and USA (19). There were 8 school teachers, 22 university staff (mathematics educators and mathematicians). Eight of these participants comprised the Klein Project Design Team.

The programme basically followed the standard AIM format: morning presentations and afternoon working sessions. For details see Appendix.1.

The Workshop led participants through an introduction to the project, the task of identifying Vignette topics, designing Vignettes, writing them, and then making critiques. During this process various ways of organising the work were tried, with group, pair, and individual combinations. Discussions during breaks focused on the Vignettes being written, as well as wider issues of the characteristics of Vignettes, formats and technical details, and dissemination methods. We also talked about potential writers for particular Vignettes or Chapters.

On Thursday evening there was a meeting of the Klein Project Design Team to discuss ongoing matters with respect to the Project.

Outcomes

Workshop Model:
The model of the Workshop was essentially successful. The gradual introduction to Klein Vignettes by thinking about topics, then about the design of a particular Vignette, then attempting to write one or supplementary materials worked on two levels. Not only did the Workshop produce a large amount of text, interactive diagrams, and reference material on a large range of Vignettes, but also it developed participants understanding of Vignettes so that they were able to hold substantive discussions about the nature of Vignettes. In addition, many participants reported significant mathematical learning for themselves.

Five days proved long, we feel that a four-day workshop would be nearly as productive and be less strain. Also we learned some procedural lessons about forming groups to write Vignettes, for example the importance of having sufficient mathematical expertise available, having a teacher (i.e. someone representing the potential audience), as well as somebody who has a concept of the Vignette as a whole. It is clear that the early stages of Vignette preparation benefit from a group dynamic, but in the latter stages pairs or individuals seemed more productive.

Vignette Characteristics:
The need for a "hook" and a mathematical "point" were reinforced. These are defining features of Vignettes. Texts that merely describe mathematical curiousities, or present a piece of mathematics on its own, do not meet the needs of the Klein Project.

Vignettes need to be short (ideally 4-6 pages) and to contain significant accessible mathematics. They must embody in some way three aspects of mathematics: contemporary developments; the many internal links within the subject; and a sense of its evolution.

A genre of writing developed in the workshop embodying these characteristics and using a style that will keep the interest of the potential audience of secondary teachers of senior classes. We now have some good models of this Klein genre.

We also began to develop criteria for making critiques, based on the Brazilian teacher workshops:

(1) Identify the motivating central problem of the text and comment its importance. Comment on whether the problem is accessible, with brief justification. Comment whether the text clarified the motivating problem.

(2) Identify the results or questioning that the article intends to transmit or to raise. Point to one or more parts of the text that attracted your attention and appreciation, with brief justification.
(3) Select the content you found more difficult to follow. Specify the type of difficuty (technical, conceptual, notation, language, etc). Comment briefly on the cause of the difficulty.
(4) Contribute suggestions that could help the text to be more accessible to the target audience. Suggest complementary texts, references, programs, websites, etc).
(5) Note connections between the text and the school curriculum: a) identify related topics of the secondary curriculum; b) comment on how the topic could be used in a secondary classroom; c) comment the validity of this article for the mathematical knowledge teachers.

Vignette Production:
Prior to the Workshop there existed nine draft vignettes in English which were posted on the website. The Brazilian Project had 5 draft vignettes (written in Portuguese) and 20 more being edited after teacher trials.

During the Workshop, one of the Portuguese vignettes was translated into English, ten further vignettes were completed to first draft form (ready to be critiqued) (two of these are in both French and English, one is in Spanish), two more are in production, and another four topics were identified and various stages of design completed. Fifteen critiques on existing vignettes were received. The complete list is given in Appendix 2.

Klein Project Community and Products:
During the Workshop it was decided to establish a Klein Project Blog and use it to build a community of critiquers and consumers of Klein Vignettes. The Blog would need a launch and dissemination through teacher and mathematician networks. Vignettes would be released gradually.

The Website will be a repository for the Vignettes and any revisions undertaken in response to feedback. It will also contain supplementary documents, diagrams, interactive items, videos, links, and references. A procedure for eventual formal Review was established.

Klein Project Development Timetable:
The Design Team decided upon the following timetable:

- To have five vignettes ready for dissemination by 31st December, 2011, and at least five more on each of 31st January, 31st March, and 30th June, 2012.
- To identify potential Chapter authors by 31st December, 2011; and have agreements in place by 28th February. Chapters to be in first draft by August, 2012.

The Design Team will meet again in the second half of 2012 in Germany, and a Workshop will be held at the same time. Further Workshops are planned in Tucson, Arizona, and in Asia in 2013.

Appendix 1: Workshop Timetable

Monday 7th November:		
am	Bill Barton	Welcome and Klein Project Introduction
am	Christiane Rousseau	Vignettes: General features, Banach, Dimension
am	Bill McCallum	Vignette: Herons Triangle
pm	Group Workshops	Identifying Vignette Topics
Tuesday 8th November:		
am	Yuriko Baldin	The Brazilian Klein Project
am	Ravi Vakil	Doodling
pm	Group Workshops	Designing a Vignette
Wednesday 9th November:		
am	Michle Artigue	The Logic Chapter
am	David Mumford	Mathematics: A Personal Overview
pm	Group Workshops	Writing a Vignette
Thursday 10th November:		
am	Writing Group Feedback	
am	Grame Cohen	Vignette: Calculators & Chebychev Polynomials
am	Hans-Georg Weigand	The Klein Website
pm	Individual work	Writing a Vignette or Supplementary Document
Friday 11th November:		
am	Individual work	Critiquing a Vignette
am	Workshop Feedback	
pm	Individual work	
Closing		

Appendix 2: Vignettes in Preparation

In draft form ready for critiquing pre-Workshop:

Banach Fixed Point Theorem
How Google Works
Herons Triangle
Tandem Rigging
Sphere Packing
Dimension
Higher Dimension
Calculators & Cheybeyshev Series
Mathematical Tidiness

In draft form ready for critiquing from Workshop:

Recurrence & Induction (French)
(Linearity & Symmetry) Struts
Curvature
The Belt Trick
Matrices & Digital Images (Portuguese & English)
Goodstein Sequences (French & English)
Fair Voting
Polynomial Roots
Rolles Theorem

In preparation:

RSA Cryptography
Shock Waves

Topic and some descriptions only:

Bezier Curves
Linkages & Rigidity
Voronoi Diagrams
+ 6 from Deedee & David Meltzer