Numerical probabilistic methods for high-dimensional problems in finance

organized by
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Workshop Summary

The main topic of the conference was the study of algorithms for solving high-dimensional optimization problems, with main emphasis on finance applications. The participants agreed to define and adopt a canonical example that will serve in the future as a test example for different methods. The example is the problem of computing the price of an American option written on the maximum of five different stocks, modeled in the standard Black-Scholes framework. In the PDE setting, this corresponds to a five-dimensional second order free boundary problem. During the third day discussion session, it was agreed that the participants will test their methods on this example, and send the software to the organizers, who will then make up a report on the numerical efficiency achieved, and post it on the workshop web page. There are three or four main approaches that will be compared. It was noted that most of the participants will meet again at a similar workshop in May 2005 at the Newton Institute at Cambridge University, during the semester on Financial Mathematics. It was then agreed that at that time there should be an evaluation of the progress made during the period between the AIM workshop and the Newton Institute workshop. During the same discussion, it was concluded that the state of the art of the current methods is the ability to solve American option type problems with up to ten dimensions. The majority of the participants is of the opinion that the same can be done for optimization problems in which only the drift of the underlying diffusion process is controlled (corresponding to quasi-linear second order PDEs), although not all the methods have been tested on this problem. As for the most difficult problem, solving the optimization problems in which both the drift and the volatility of the underlying diffusion process are controlled (corresponding to fully nonlinear second order PDEs), the prevailing opinion was that no methods are presently successful in more than two or three dimensions. Thus, the most ambitious task that the participants will try to perform in the future is to improve current methods, or come up with new methods to be able to deal with higher dimensions in this problem. It was realized during the conference that there is a possibility of combining the different approaches presented at the workshop, and new collaborations were started with this in mind. In particular, this lead to initiating new collaborations between researchers in the U.S. on one hand, and France and U.K. on the other hand. During the workshop it became clear that these are natural and desirable collaborations in order to improve the state of the art, because of the different methods being efficient at different aspects of the problem. This hardly would have been possible if it had not been for the specific structure of the workshop, namely a small group of researchers focused on a specific problem, and with a lot of time for interaction. The impression is that everyone was delighted by this format and by how much progress was made because of it. Let us also report on interactions not directly related to the main topic of the conference. The first day we had a general discussion on when we should worry about high-dimensions, when

it is possible to reduce the number of dimensions, or when we should actually approximate the desired setting by higher, possibly infinite dimensional models, in order to have more tractability. The second day discussion was led by Darrell Duffie of Stanford Business School, in which he presented one of the biggest computational challenges in the financial industry today: statistical estimation and numerical simulation of models for default (bankruptcy) times of a large number of companies whose stocks are traded in the financial market. Typical number of companies in these computations, equal to the dimension of the problem, is between fifty and five thousand. This is much higher than in the problems of the previous paragraph, but there is no optimization, thus the problem is linear, but still computationally very challenging. Several participants were involved in the discussion, and there were three or four potentially promising suggestions, especially by the U.K. participants Chris Rogers and Terry Lyons, which is likely to lead to a new collaboration with D. Duffie. Another immediate progress has been made in a problem presented by Vicky Henderson (Princeton University): in her setting, the problem of computing the price of an option in a model with stochastic volatility is equivalent to solving a Backward Stochastic Differential Equation, for which her co-authors and her did not know if there is existence and uniqueness theory. Jin Ma (Purdue University) stated that there are recent results on this under conditions that seem to be satisfied in her problem. In short, the workshop provided a perfect setting for an impetus to several new collaborative efforts, some expected and some unexpected. Perhaps most importantly, it went a long way in forming a general agreement on what is the most important and challenging problem in this field, what the state of the art is of the existing methods, and what directions to take in order to improve the state of the art.