Mathematical and Geophysical Fluid Dynamics

organized by
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Workshop Summary

Deterministic and stochastic aspects of Fluid Dynamics, in particular the phenomenon of turbulence, are of great importance in many fields of engineering and science. It presents some of the most difficult problems both in fundamental understanding of its nature and in applications, many of which are still unresolved. Mathematics and the geosciences have already established fruitful interaction through the famous Euler, Navier-Stokes, and advection-diffusion equations and their geophysical counterparts, the Boussinesq and Primitive Equations of the oceans and the atmosphere. These equations are at the core of the General Circulation Models (GCMs) of the atmosphere, the ocean, and coupled atmosphere-ocean models, while probabilistic versions of these equations provide models for turbulent behavior.

The workshop was designed to facilitate transfer of recent advances in mathematical and statistical Fluid Dynamics to the geophysical community involved in the modeling of the ocean and atmosphere and also to introduce the mathematically oriented participants to new trends and problems in geosciences related to the treatment of large scale models of climate and related subsystems.

In the last decade, a substantial progress has been made in several new areas of fluid dynamics including, but not limited to, the following:

(1) The mathematical theory of the equations of the atmosphere and the ocean has been developed for such equations as the Boussinesq and Primitive Equations (PEs) and related equations.
(2) The connection between the mathematical theory of the Navier-Stokes equations, the dynamical systems theory, and the theory of turbulence has advanced in several directions including attractors and inertial manifolds.
(3) New approaches to fluid dynamics were established based on stochastic equations of fluid dynamics. These include stochastic Lagrangian models of fluids and stochastic vortex filament based models; applications of Wiener chaos expansions to the decoupling of the Reynolds equation and computing statistical moments. In addition, some stochastic versions of the Primitive Equations have proved to be instrumental in modeling the ocean/atmosphere interaction that is the key to understanding climate variability.
(4) Within geophysical modeling and prediction, the necessity of the inclusion of stochastic physics has been recognized as a missing aspect in climate simulation, weather uncertainty prediction and in the closure of inhomogeneous geophysical turbulence. These developments have uncovered new problems in the numerical solution and diagnostic analysis and verification of model simulations and predictions.

Many of these issues were addressed by the Workshop.

The participants represented three scientific communities: geociences, mathematical fluid dynamics, and stochastics processes. While, all three groups have common interest in fluid dynamics and its applications, their approaches, techniques, and the scientific lingo are quite different. This presented a substantial challenge for the organizers. It turned out that the lack of a rigid program characteristic for AIM conferences was ideal for the workshop that consisted of a few formal presentations and many free flowing discussions.

A number of world renowned experts presented their current research at the Workshop. We highlight a few illustrative examples:

(1) A general discussion on physically based atmospheric models was led by J. Tribbia and R. Buizza.
(2) B. Rozovsky (USC) outlined some mathematical approaches to the stochastic backscattering in Large Eddy Simulations. In particular it was proposed to model effects of high wave-number modes by a random field with the asymptotic energy spectrum $\sim k^{-5/3}$.
(3) Leonid Piterbarg (USC) presented a review of recent results on Lagrangian stochastic models (LSM) and their application to oceanographic problems. Several open problems were suggested by the participants: developing a mixing and predictability theories in the inhomogeneous case and testing the suggested data assimilation schemes in OGCMs
(4) Claudia Pasquero (CalTech) introduced a new statistical model of heterogeneous convection in the ocean.
(5) Cecile Penland (NOAA) has discussed a hierarchy of probabilistic models practically useful for geophysical applications. (I am unable to elaborate on this talk).
(6) Anna Amirrdjanova (MSU) spoke about the stochastic vorticity equation.
(7) Susan Friedlander (UCI) presented some new cutting edge results concerning the stability of equations of fluid dynamics. There was a long and animated discussion on the relation of the presented mathematical results to the practical needs of geophysics. Afterwards, some of the participants from geophysical community (e.g. R. Buizza) pointed out that this discussion was very helpful to them.
(8) Judith Berner presented one of the industrial weather forecast models. One of the interesting features of this model was a cellular automata based algorithm for modeling clouds.
(9) A discussion of the mathematical theory of the Primitive Equations was led by Roger Temam: existence and uniqueness of solutions, attractors, inertial manifolds. Detailed presentations in these directions were made by Mohammed Ziane, Madalina Petcu, Antoine Rousseau and Ning Ju.

Among the many things learned and the many ideas exchanged in this workshop, we would like to emphasize two points which, we believe, have an important potential for future interactions and developments:

- the mathematical part of the participants had a rather negative opinion on perspective of applications of cellular automata in fluids because of unreasonable previous attempts to use them at a very large scale, in another area. However, at this workshop, geophysicists from ECMWF have presented a less ambitious and practically important use of cellular automata for modeling the dynamics of clouds. A number of mathematicians have found this development interesting and promising, and this adds to another application encountered elsewhere. It would be useful to look into these applications in more detail, as well as other applications. We believe that such applications could be the object of a workshop by itself.

- In the more specific area of the workshop, stochastic modeling of geophysical fluid flows, the repeated statements concerning "the physics to be added to the model" should be explained in more details to the mathematicians and explored. This is an area in which progress in formulation of problems if not in their solution can be raised during a workshop, in accordance with the philosophy of AIM. E-mail exchanges after the workshop show a great potential of applications.