Geometric perspectives in mathematical quantum field theory

organized by
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Workshop Summary

The mathematical construction and analysis of quantum fields remain central challenges in mathematical physics. Of special interest are quantum fields of geometric nature. Over the last twenty years the community has become seriously fragmented, with new ideas generated on mutliple fronts but with little or no communication between these subcommunities. The major accomplishments of this workshop were:

(i) brought together researchers active in various aspects of quantum field theory and greatly increased mutual understanding of different objectives and methods;
(ii) introduced new researchers, and those with mathematical expertise relevant to quantum field theory, to the central ideas, challenges, and accomplishments in the field.

Activities

The workshop began on Monday morning with a grand overview of the subject presented by Leonard Gross, whose experience in the field spans over five decades. He traced out the connections between ideas of interest to to each of the participants. Following this, Bruce Driver presented a detailed account of the Wess-Zumino-Witten (WZW) model, and shared many insights not available in publications. The afternoon saw vigorous discussions, in which a variety of questions were brought up and fielded by Rajeev, Reshetikhin, and Pickrell. The relationship with representations of loop groups was explained in cross-discussions between Pickrell, Rajeev and others. The entire discussion was a unique experience, revealing the behind-the-scenes thinking and ideas that one cannot find in published works.

On Tuesday Rajeev gave a talk explaining ideas of renormalization, connecting back to classical ideas in mathematics. He discussed important determinants that appear in quantum field theories and how they can be regularized. He also explained cut-offs and their role in infinite dimensional integrals. Problems were identified. The second talk, by Thierry Lévy, gave an exposition of the large-N limit of Yang-Mills quantum theory in two dimensions, the master field and certain differential equations, called the Migdal-Makeenko differential equations, that arise in this study. In the afternoon there were breakout sessions.

On Wednesday, Nicolai Reshetikhin gave an elegant account of classical field theory explaining a powerful approach that was not known to most members of the audience. He illustrated the power of this formalism by working out Yang-Mills theory in these terms as well as Chern-Simons field theory. Following this, Jonathan Weitsman explained his powerful program of using measures on Banach manifolds to describe quantum field theoretic and topological quantities of great interest. He identified several specific problems that were of interest to several more probability-oriented experts in the workshop.

Thursday's talks and discussions focused on the Connes-Kreimer renormalization technology, presented by Matilde Marcolli, a leading expert in the area. Lisa Jeffrey presented a precise exposition of Chern-Simons theory and related ideas. A formal exposition of problems and techniques involving the Ricci curvature was presented by Artem Pulemotov. Johnson-Freyd and Nguyen also explained their ideas relating to quantization in discussion groups.

Friday was devoted to smaller sessions and discussion groups.

There were a large number of discussion and breakout sessions all through the week. Problem and discussion groups included those led by (i) Paycha, Abdelmalek, and Marcolli on renormalization; (ii) Rajeev, Pickrell, Gordina, Driver, and Gross on loop groups and other renormalization problems; (iii) Rogers on BRST quantization; (iv) Lévy, Hall, Kemp, Kar, and Sengupta on large-N behavior of Yang-Mills. Notes were prepared, especially by Johnson-Freyd, and shared with participants.

The organizers were active participants in the talks and in discussion sessions, and worked to improve exchanges between different groups of researchers among the participants.

Concluding remarks

This was the first working meeting in many years to bring together many different strains of mathematical quantum field theory to yield a coherent picture to the participants. Individual participants gained and shared important insights into a variety of specific problems they are working on. The foundation was laid for a more coherent research community in the subject, and future meetings are being planned.