

# EXTREME FORMS OF REAL ALGEBRAIC VARIETIES

organized by

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## Workshop Summary

The workshop was centered around the question “*What is the best real form of an algebraic variety.*” This question is answered for planar curves and, during the workshop (mostly during the discussion section), conjectured answers were given for some higher-dimensional cases. Here is the list of talks.

Thursday April 6, 2006

- In the morning: O. Viro (introductory talk on Harnack curves and surfaces) and I. Itenberg (introductory talk on combinatorial patchworking).
- In the afternoon: J. Solomon (talk followed by a discussion on real analogs of Gromov-Witten invariants).

Friday April 7, 2006

- In the morning: N. Reshetikhin and M. Gross.
- In the afternoon: discussion on Harnack varieties.

Saturday April 8, 2006

- In the morning: E. Ionel and Y. Soibelman.
- In the afternoon: N. Reshetikhin and G. Mikhalkin (short talk followed by a discussion on space Harnack and tropical curves).

Sunday April 9, 2006

- In the morning: I. Zharkov and B. Parker.
- In the afternoon: D. Speyer (short talk followed by a discussion on anti-Harnack curves).

One of the central topics of the workshop was the study of a class of extremal real curves called simple Harnack curves as well as their higher dimensional generalizations. Since the one-dimensional Harnack varieties are relatively well-understood, one of the main goals of the workshop was an attempt to study the higher-dimensional case, first of all trying to find appropriate definitions and formulate problems. This was the principal purpose of the discussion that took place Friday afternoon. On Thursday, as a preparation for the discussion, O. Viro and I. Itenberg gave introductory talks on Harnack curves and the combinatorial patchworking.

The discussion was very successful and allowed the participants to crystalize possible definitions of Harnack varieties of dimension  $\geq 2$ . These definitions were mutually compared and studied in details during the discussion. Several open problems on Harnack varieties, and especially Harnack surfaces, were formulated. At the end, the discussion turned towards

so-called *anti-Harnack* or *hyperbolic* curves (the curves which from the point of view of geometry of real algebraic curves are opposite to Harnack curves). A list of problems on the structure of these curves was given as well.

Another major topic of the workshop was the problem of generalization of the real curve count (given in the zero genus case by J.-Y. Welschinger) to higher genera. The Welschinger invariants are designed to bound from below the number of real rational curves which pass through a given real generic collection of points on a real rational surface. These invariants can be seen as real analogs of genus zero Gromov-Witten invariants. An open intriguing problem is to find invariants in other situations, and, in particular, to generalize the Welschinger invariants to positive genera as well as to higher dimensions. These possibilities (from a symplectic point of view) were addressed in the talks by J. Solomon and E. Ionel.

One of the most recent field for applications of Real Algebraic Geometry is Statistical Physics. There Harnack curves (more precisely, their amoebas) appeared after the work of R. Kenyon and A. Okounkov as limit shapes of the height function in the Dimer model. The talk of N. Reshetikhin clarified this connection.

Overall, as the conference participants came from different (and quite diverse!) areas of Mathematics, the workshop benefited from the interdisciplinary interactions and opened new perspectives in the study of Harnack varieties.