

VOLUME ENTROPY RIGIDITY

organized by

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Workshop Summary

The workshop aimed at gathering researchers from different backgrounds around the theme of volume entropy and rigidity. In 1995, Gérard Besson, Gilles Courtois and Sylvestre Gallot invented a geometric technique which allowed them to give a purely geometric proof of Mostow rigidity theorem for locally symmetric spaces of negative curvature. Volume entropy, the exponential growth rate of the volume of balls in the corresponding symmetric space, is a numerical invariant which plays a key role in their proof. There are famous challenges which have been the subject of research after BCG. Besides finding new applications, major problems are to generalize the BCG techniques to higher rank locally symmetric manifolds, to singular spaces or to other geometries such as Finsler geometry, for instance. Each of these generalizations would bring a new approach on known problems and has been explored by different specialists. One of the workshop's goals was to gather these specialists and to let them confront their experiences and their initiatives.

There was no major breakthrough during the workshop and this was expected. Nevertheless, the general impression of the participants was extremely positive. Many of us discovered problems and ideas that were close to our own and therefore could become sources of fruitful interactions. Following the AIM format, the workshop was divided into morning talks and afternoon more informal brain-twistings. The morning talks were diverse, all around the theme of volume entropy and rigidity, and the speakers made the effort to be understandable by all the audience, who were supposed to be familiar only with the core of BCG's argument.

Gérard Besson, Gilles Courtois and Sylvestre Gallot each gave a talk. Gérard Besson exposed a connection between volume entropy and the Ricci flow. This was a demand of the organizers that Gérard accepted. After the workshop, he wrote us that his talk and the reactions to it opened him to possible new developments. Gilles Courtois spoke about the use of BCG techniques to provide a geometric approach for the property of uniform exponential growth of groups and manifolds and about the fascinating possible link with the Lehmer problem in number theory. Sylvestre Gallot gave a talk on the very recent work of his student Luca Sabatini towards the developments of BCG in singular spaces. This approach is particularly adapted to rigidity problems on buildings and other combinatorial objects. Bertrand Rémy gave a clear introduction to buildings and recent developments on their cohomology, lattices and boundaries. Following Bertrand's talk, one of the afternoon groups compared different approaches to computation of volume entropy and other rigidity problems on hyperbolic and Euclidean buildings. The talk by Tatiana Nagnibeda gave another perspective, with several results about the volume entropy of graphs and the related properties of the Culler-Vogtmann Outer space.

The other themes of the workshop followed the same scheme as the building session: they were introduced by a general talk in the morning, followed by discussions in smaller groups in the afternoon. The participants to these groups might still have very different backgrounds and the informal format allowed explorations and clarifications that we found very useful. Particularly active was the group about simplicial volume. After a first short introductory talk by Roman Sauer and the general presentation by Michelle Bucher on Milnor's inequality, the group met every afternoon. It studied the relations between foliated simplicial volume, L^2 Betti numbers, minimal volume entropy and bounded cohomology. Andrea Sambusetti explained several constructions of counterexamples to natural conjectures. A more precise statement of the main topics in their discussions can be found in Roman Sauer's report.

Other groups discussed examples of variable curvature (after the talk by Andrea Sambusetti on the discrepancy between volume entropy and the critical exponent in some non-compact finite volume manifold of pinched negative curvature) and the analogous problems linked with the volume entropy in Hilbert geometries (after the joint talk by Constantin Vernicos and Mickael Crampon). Actually, those two groups kept talking to each other and realized that their problems were very close, that related pathologies might occur. Since Hilbert geometries are a particular class of Finsler manifolds, the question about more general Finsler manifolds naturally turned up. Patrick Foulon remarked that for irreversible Finsler manifolds the volume entropy should be replaced by the cohomological pressure. In one of the afternoon sessions Patrick explained this notion and how it might be pertinent to several problems which – in the Riemannian setting – are formulated in terms of the volume entropy or topological entropy. Françoise Dal'bo's talk about the volume entropy of subgroups came later in the week and the working group was slightly less energetic, but it was a place of rich exchanges and one of the participants expressed to us how the talk and the discussions offered her new perspectives on a classical subject. Pete Storm gave a talk on BCG theorem for finite volume manifolds, a topic that many participants were interested in. Following his talk, an afternoon session was about possible difficulties of the BCG method in higher rank locally symmetric manifolds. There was not much progress in this direction.

The general talks of the last day were more technical: Inkang Kim explained that the Toledo invariant is not linked with rigidity. Frédéric Paulin gave a new application of Ergodic Theory to equidistribution problems in number theory. It is a mark of the success of this conference that these two rather difficult presentations were followed and understood by the majority of participants, in spite of the variety of backgrounds. The workshop finished by a problem session which is reported elsewhere.

There was not much progress in the BCG method in higher rank and the case of buildings, which were our original possible goals. However, many possible new paths were discussed, especially the BCG method for $CAT(-1)$ -spaces, volume entropy and Ricci flow, possible developments in simplicial volume, growths of subgroups, and appropriate questions in Finsler manifolds.

Confirmed by the comments of the participants after the workshop, we believe that the workshop was very successful. The format of the AIM workshops were new to many participants, and at the end of the workshop, most of them were enthusiastic about it. There were some remarks on whether a non-prepared informal presentation (in the afternoon) is desirable or not, but most of the participants appreciated the discussion sessions in the afternoon.

We gratefully acknowledge financial support from both the American Institute of Mathematics and the “Forschungsinstitut für Mathematik” at the ETH Zurich. In particular we want to thank Marc Burger as director of the FIM for his enthusiasm about organizing this workshop in Zurich and for the hospitality we received at the FIM. We also want to thank the AIM staff for their expert help in making this meeting an interesting and rewarding experience.