Eisenstein Series and Applications

organized by
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Workshop Summary

The goal of the workshop was to discuss applications of Eisenstein series to problems in arithmetic geometry and number theory.

Eisenstein series are an important ingredient in the theory of automorphic forms and L-functions. They have also been exploited extensively by number theorists for many other arithmetic purposes; for example, the Fourier coefficients of Eisenstein series encode a significant amount of arithmetic information. We had brought together specialists working on different facets of this theory with the goal of stimulating the flow of ideas between these different groups.

The workshop focused on 3 areas in which Eisenstein series have played a central role:

- Derivatives of Eisenstein series and Arakelov intersection theory on Shimura varieties (the work ofKudla-Rapoport-Yang).
- Special values of L-functions and Iwasawa theory (the work of Skinner-Urban).
- Equidistribution of rational/integer points on homogeneous varieties (the work of Shalika-Takloo-Bighash-Tschinkel).

Correspondingly, there were 3 main lecture courses with introductory lectures, as well as discussion sessions featuring explanations of concrete examples and research problems. In addition, we had organized talks on Eisenstein cohomology, in particular, on recent interesting work of Mahnkopf (Speh and Shahidi), as well as talks on p-adic aspects of Eisenstein series (Skinner and Emerton), recognizing the growing importance of p-adic techniques in the theory of automorphicforms.

The organizers feel that the conference was very successful. We have had very positive feedback from participants, and several mathematical projects have advanced as a direct result of this meeting. For example, discussions between Takloo-Bighash and Lapid lead to a resolution of an obstactle in the proof of analytic properties of height zeta functions and the completion of the the paper by Shalika, Takloo-Bighash and Tschinkel (math.NT/0602236) "Rational points on compactifications of semi-simple algebraic groups". During the conference, Gan and Kudla began to discuss the metaplectic cover an anisotropic form of SL(2). They did a preliminary survey about what is known about its cocycle via the AIM library and have continued the discussion about carrying over the results of Labesse-Langlands to this case via the trace formula. Following up on the interactions in small working sessions at the conference, Lapid proved meromorphic continuation of Eisenstein series for smooth, not-necessarily K-finite, sections. This is very nice result and will be very useful in practice. The paper will appear in the proceedings.

A spectacular recent development (March 2006), not directly related to the activity at our workshop, is the proof of the Sato-Tate conjecture by Taylor, via p-adic lifting techniques

and the proof of automorphy of L-functions of symmetric powers of the motive of an elliptic curve. Previous special cases (e.g., symmetric cube) were obtained by Shahidi and Kim via the analysis of constant terms of Eisenstein series of exceptional groups (the Langlands-Shahidi method).

We are currently preparing a volume "Eisenstein series and applications", under contract by Birkhauser, Progress in Math. series, which will include survey and research papers by: Bump, Emerton, Franke, Gan, Harris, Lapid, Kudla, Prasanna, Shahidi, Skinner and Takloo-Bighash.

We greatly appreciate the support we had from our hosts at the American Institute of Mathematics: B. Conrey, H. Moore and D. Farmer. Their advice helped shape the structure of the workshop, as we were preparing it, and was clearly beneficial for the dynamics of mathematical interactions at the meeting.