Deformation theory, patching, quadratic forms, and the Brauer group

organized by
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Workshop Summary

The AIM workshop on deformation theory, patching, quadratic forms, and the Brauer group was held on January 17-21, 2011. Participants came from a variety of institutions in the USA, Europe, and India, and ranged from graduate students to eminent senior faculty. Activities were focused on two main problems: the period-index problem for the Brauer group and the u-invariant problem for quadratic forms. These problems have experienced dramatic progress in recent years due to the application of novel geometric, cohomological, topological, and algebraic techniques by a variety of researchers coming from algebraic geometry, arithmetic, pure algebra, and K-theory. The workshop brought together researchers in all of these areas, resulting in a lively exchange of ideas, some progress on these problems, and a number of new and interesting questions.

Each morning of the workshop featured two lectures: Krashen and Merkurjev on Monday, Harbater and Lieblich on Tuesday, Parimala and Poonen on Wednesday, Saltman and Colliot-Thélène on Thursday, and de Jong and Kahn on Friday. The first two days were expositions of the problems and techniques that have been used, the third day focused on relations with local-to-global problems, and the last two days concerned algebraic geometry, cohomology, and K-theory as they relate to the principal problems of the workshop.

The afternoons consisted of problem sessions (Monday and Wednesday) and small group work each day. The small groups were devoted to a range of topics including 1) refining the period-index conjecture for higher cohomological degrees with small coefficient sheaves, 2) studying the u-invariant of function fields of curves over number fields, 3) studying finiteness properties of the u-invariant for extensions of fields with finitely many square classes, 4) showing that the Brauer group and cohomological group are distinct, 4) proving cyclicity for classes in the Brauer groups of abelian varieties. These topics were decided upon by group votes each day and were accompanied by daily progress reports.

By the end of the workshop, a nice collection of problems and conjectures had been assembled. A pleasing general period-index conjecture for mod ℓ coefficients was formulated, and the Colliot-Thélène conjecture was tied to the u-invariant of $\mathbf{Q}(t)$ by mixing twisted sheaves with the methods of Parimala and Suresh. We expect that several of the discussions at the workshop will lead to published papers.