Holomorphic curves in contact geometry

organized by
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Workshop Summary

Since work of Gromov in the 1980's holomorphic curves have been an essential tool in the study of symplectic manifolds. More recently they have become a central tool in contact geometry and low dimensional topology. The links this suggested between these subjects is novel and exciting. The goal of this conference was to begin the process of unifying the diverse ways in which holomorphic curves are being used. The conference was highly successful at bringing together a diverse group of international experts, outlining the main ways in which holomorphic tools are currently being used, and discussing several potential approaches for their unification and further application. The following trends were emphasized:

Legendrian Contact Homology and Invariants of Topological Knots: On this topic Ekholm and Traynor discussing contact homology in jet spaces and Ng discussing the construction of invariants of knots in $\mathbb{R}^3$ from contact homology. This approach seems to be related to the one suggested by Ooguri and Vafa, which was outlined at the conference by Grassi. There was much discussion of potential links between the two approaches which we hope will stimulate further research in this area. Significant progress was also made in building the foundation of contact homology needed for Ng's work.

Computations and Applications of Contact Homology: Giroux and Auroux discussed open book decompositions of contact manifolds and Lefschetz pencils of symplectic manifolds and ways these topological objects could be used in the computation of contact homology. Bourgeois discussed using contact homology to detect non-trivial elements in the homotopy groups of the space of contact structures/contactomorphism group.

Heegaard Floer Homology of 3-Manifolds: On this topic we had Ozsvath discuss applications of this new invariant to contact geometry and give connections with Khovanov's Homology. Also, Stipsicz discussed using these invariants to answer various questions concerning the types of contact structures a manifolds can support.

Embedded Contact Homology: Hutchings discussed a homologyfor mapping tori of surfaces that is somewhat similar to contact homology but used embedded holomorphic curves instead of immersed ones. He also described a computation he made with Sullivan of this homology for T^3.

Related fields where holomorphic techniques could provide insight: Here we had many experts describing problems that could be related to, and possibly solved by, the techniques discussed during the workshop. Among these speakers we had: Akahori (CR geometry), Mitsumatsu (Foliation theory), Honda (Legendrian and transverse knots) and Biran, McDuff, and Polterovich (Lagrangian submanifolds).

During numerous discussion sessions, programs were outlined that could connect Seiberg-Witten Theory and Heegaard Floer Homology (discussion lead by Lee) and Heegaard Floer

Homology and some enhanced form of symplectic filed theory (discussion lead by Eliashberg). There were also more informal discussions about surgery formulas for contact homology and relations with open book decompositions.

The informal atmosphere of the workshop encouraged a great deal of discussion among the participants. This lead to new collaborations (e.g. Bourgeois and Yau began work on a joint paper on contact homology of sub-critical manifolds), the continuation of ongoing collaborations (e.g. Ozbagci and Stipsicz; Eliashberg and Polterovich; Ekholm, Etnyre and Sullivan as well as a new collaboration between this group and Zhu) the solution to open problems during discussion sessions (e.g. a question of Polterovich about weakly boundary rigid tori was answered by Eliashberg) and a much better understanding of the current state of various research projects by all attendees. Many participants contributed to problem sessions. Several new very stimulating problems were formulated. They will be posted online and will undoubtedly serve as a source of new exciting results in the area.