

# ARITHMETIC HARMONIC ANALYSIS ON CHARACTER AND QUIVER VARIETIES

organized by

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## Workshop Summary

The goal of this workshop was to bring together a group of mathematicians who work in one of the four main themes of the workshop: cohomology of character and quiver varieties, applications of the Weil conjectures to cohomological calculations, representation theory of finite groups and algebras of Lie type, and combinatorics of Macdonald polynomials. These four subjects come together in the organizers recent work, (a draft version of the paper was distributed among the participants of the conference in the first day of the workshop). The main focus of the workshop was to discuss these subjects on their own and then explore existing and possibly new connections between them.

The first two days of the workshop were devoted to introductory lectures on the main four subjects. The talks included introductory talks by Nakajima, Katz, Lehrer and Haiman in the respective fields. The first day was entirely dedicated to the introductory talks and the many questions which they generated. The second day saw some more interactive activities in the afternoon after the two introductory lectures in the morning. Tuesday's afternoon started with one hour groups sessions on specific questions stemming from the introductory talks. In the rest of the afternoon, the workshop separated into two parallel sessions one discussing Mixed Hodge Structures with Katz, the other learning recent work on Hall algebras of elliptic curves, a recent very exciting subject, from the co-inventor Schiffmann.

Wednesday was devoted to talks by the organizers where they discussed their perspectives on their joint work. Hausel concentrated on giving an idea of the main motivation for the collaboration, which ultimately comes from 4-dimensional supersymmetry theories in Physics. Villegas explained the combinatorics tools behind the main conjecture and results: symmetric functions, Möbius inversion in the set partition poset and their relation to the character theory of the general linear group over a finite field. Letellier discussed the representation theoretic aspects and implications: connections between the character theory of a group and its Lie algebra over a finite field and the existence of a common two variable deformation of both.

On Thursday morning two more specialized talks took place, one by Crawley-Boevey who gave an elementary introduction to his recent solution of both (the additive and multiplicative) Deligne-Simpson problems; the lecture was organized around the idea of reflection functors. Bergeron then gave a very down to earth "How to calculate with Macdonald polynomial" talk which was very useful for people (like the organizers) who are interested in calculating with Macdonald polynomials explicitly.

The afternoon saw a very successful open problem session moderated by Ben-Zvi, which resulted in a dynamic 90 minutes discussion on open problems on ideas connected to the workshop topics. Later in the afternoon (continuing to as late as 7.15 for one of the groups) two groups formed. One, followed a 3 hour highly interactive lecture by Katz on middle

convolution (which saw an exciting discussion between Crawley-Boevey and Katz discussing the connection between middle convolution and reflection functors); another, continuing a discussion by Schiffmann of his exciting recent work.

Friday was concluded by Schiffmann explaining his recent work in very detailed and ground to earth manner to all participants. Ram gave the final talk which connected the representation theoretic aspects with combinatorics. The afternoon was devoted to wrap-up discussions about the ideas which appeared in the workshop and the planning of future activities related to the workshop. In particular, the idea arose of having a blog, which could serve as a meeting point to continue discussions and keeping up to date on developments. The blog was indeed started and can be found at:

`frvillegas.wordpress.com`

The workshop seemed to generate a lot of excitement about the connections between the various subjects. Many of the participants were particularly interested in the work of the organizers contributing many questions and suggestions for further research. Apart from the many different subjects everyone could learn about in the workshop, many dynamic discussions arose among participants, which could potentially lead to further collaborations.

Examples of the new potentially interesting ideas: Hausel learned about the multiplicative quiver varieties constructed by Nakajima's student, which seem to connect back to Nakajima's own additive quiver variety story in an interesting way. It was suggested to Hausel by Oblomkov to study particular quivers (in particular the tennis-raquet graph) which arose naturally in the study of Double affine Hecke algebras. Crawley-Boevey mentioned to Hausel that it is not known if all character varieties are connected (a result which he would need in his theory of the representations of multiplicative preprojective algebras). This should be fairly easy to check using the organizers's main result as was done in the first paper of Hausel and Villegas.

Nakajima suggested to study various extra symmetries of Macdonald polynomials to see what the effect is on the conjectured mixed Hodge polynomials of character varieties. Haiman was thinking about the specialization of the conjecture of the organizers on mixed Hodge polynomials of character varieties, which should yield the Poincare polynomial. The straightforward specialization of the conjecture is not working because of the appearance of various poles, but Haiman's intuition on Macdonald polynomials make him suspect some roundabout way. Finally Hausel learned about affine Grassmannians from Nadler, and compactifications of certain spaces of cluster algebras from Webster, both subjects could lead to some potentially fruitful connection with the existing literature.

Villegas is quite keen on pursuing the connection between the abstract formulation of middle convolution by Katz and the more down-to-earth one of Crawley-Boevey in terms of quivers. There seems to be a big open space there to be explored. In particular, it would be nice to understand what is the algebra generated by the basic operations defined by Katz (tensor product and middle convolution on the projective line). One would expect that the recent matrix formulations of middle convolution should make it possible to calculate things very explicitly.