# Calibrations

organized by
Robert Bryant, Xiaobo Liu, and Pit-Mann Wong

Workshop Summary

The format as set up by AIM received universal endorsements from the participants. A typical response from the participants being "the format and the environment at AIM are very conducive to exchange of ideas and to making progress in my work". The workshop was the first on 'Calibrations'—an area that is attracting more and more attention of mathematicians in different areas and some physicists. The outcome of the workshop well exceeds the expectation of the organizers. We are particularly impressed by the contribution made by a number of young mathematicians and one young physicist during the workshop. We are also very pleased that quite a number of collaborations were formed during the workshop. It is our sincere hope that the participants will continue to build upon the success of the workshop and others will join us through the new webpage on Calibrations to be developed by AIM and the participants.

## 1. New Collaborations

a. Dominic Joyce and Ilarion Melnikov: Homological Mirror Symmetry. Hopefully, Melnikov will go to Oxford as a postdoc.

b. Jaeh Youk Lee is working with S. Karigiannis and (maybe) M. Ionel on "Analogues of symplectic reduction on $G_2$- and Spin(7)-manifolds".

c. Likely collaboration of Dan Fox with Ian McIntosh on using integrable systems to study 2-ruled Cayley and coassociative cones (in $\mathbb{R}^8$ and $\mathbb{R}^7$ respectively) with 2-tori links. This may overlap with work by Carberry and Wang, so there may be a collaboration among all four people. A tentative collaboration was mooted with Daniel Fox, Ian McIntosh and Erxiao Wang. They are interested in first giving an integrable systems approach to coassociative cones over tori (describing them in terms of algebro-geometric data—this is strongly related to an existing project of Emma Carberry with Erxiao Wang) and then moving on to the Cayley case.

d. Daniel Fox and Ilarion Melnikov have begun to discuss some mirror-symmetry like structures for Spin(7)-manifolds. The hope is to at least understand the subtleties involved in an $SYZ$-like argument.

e. Conan Leung and S. Karigiannis will be working on the extra structures on the moduli spaces of associatives and coassociatives in a $G_2$-manifold.

## 2. New Directions

a. Jaeh Youk Lee proposed a few possible models (e.g., vector cross products), of analogues of symplectic reductions on $G_2$- and Spin(7)-manifolds.

b. Ian proposed to investigate how spectral curve genus is related to the stability index of special Lagrangian cones. A similar question can be asked in any situation where singularities of calibrated manifolds are related to integrable systems.

c. There might be some interest in studying special Lagrangian geometry in elliptically- or $K3$-fibered Calabi-Yau 3-folds. It was pointed out by Joyce and Tian that this probably is only interesting in the limit of small fibres, but it still seems like it might be an interesting structure.

d. The introduction of $\phi$-plurisubharmonicity in calibrated geometry by Lawson and Harvey is an interesting new direction. Up to now, study of calibrations has been almost exclusively geometry, and this new analytic approach is exciting.

e. Dominic Joyce's discussion of difficult technical issues involved in the study of singularities of special Lagrangian pointed out several directions that should be pursued.

## 3. New Developments

a. In discussions with to Mark Haskins, Joyce came up with a method for constructing examples of Special Lagrangian 3-folds with conical singularities in compact Calabi-Yau 3-folds, and a method for constructing a compact Spin(7)-manifold with a Cayley fibration. Both still need lots of work.

b. Seungho Wang became interested in special Legendrian surfaces after talking with Mark Haskins and learning more about Haskins' constructions with Kapouleas.

c. Dan Fox's work on 2-ruled cones points to a wider application of Riemannian twistor theory to understanding calibrated submanifolds in Euclidean spaces.

d. Melnikov is very interested in Joyce's idea to use the unobstructed Floer homology as an additional condition on 'physically reasonable' sLag cycles. First, given what we have seen at the workshop about constructions of sLag cones, a complete classification of singular sLags appears hopeless. Second, this condition has a natural physical interpretation, and it would be extremely interesting if it restricted the sLag geometry so as to make it manageable.

## 4. Visions For The Future?

a. Lee forsees the possibility of obtaining 4- or 3-manifolds with special structures. And hopefully, this will give a way to see mirror symmetry on $G_2$- and Spin(7)-manifolds.

b. Ian McIntosh believes there could be some progress in understanding associative, coassociative and Cayley submanifolds as higher dimensional versions of $J$-holomorphic curves, and developing a sort of twistor theory for these, in the same way that many examples of minimal surfaces in homogeneous spaces can be constructed from $J$-holomorphic curves in related twistor spaces.

c. Karigiannis feels strongly that the understanding of singularities of calibrated submanifolds, in particular the types of singularities which can arise as singular fibres of fibrations by calibrated submanifolds will be of fundamental importance.

d. Another vision of the future is a better understanding of the role and importance of spinors in calibrated geometry and special holonomy.