The Caccetta-Haggkvist conjecture

organized by
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Workshop Summary

This is a report of the recent workshop held at AIM on the Caccetta-Haggkvist conjecture. The basic question and its ramifications are described in the accompanying text by Blair Sullivan, and here we describe only the events of the workshop.

We began with several talks describing the current state of the conjecture. Sullivan started by surveying known results and old open problems related to the conjecture; Seymour gave a talk on his favourite variations and strengthenings; and Shen spoke on his results, which are the best partial results known. Thomasse talked about the problem from his point of view, highlighting several challenging special cases. Devos described the connections between the conjecture and additive number theory, and later presented his ideas to apply game theory to the conjecture. Mader surveyed his theorems on graph connectivity, and Nathanson gave us a tutorial on additive number theory. Fianlly, Bondy sketched his old proof method of applying linear algebra to the problem, counting different types of subgraph. These talks were distributed over the week, but with more talks towards the start.

Mostly in the afternoons, we had discussion groups and working sessions focussing on special topics. One on a new conjecture, due to Chudnovsky, Seymour and Sullivan (we didn't solve it, but Kostochka had a partial result that he spoke about, and Sudakov had an interesting idea and technique to obtain a partial result, that occupied us for several hours); one on a possible generalization of certain additive number theory results (this turned out to be false); one on attempts to construct counterexamples; one on the Hoang-Reed conjecture (here Mader gave us a counterexample to a strengthening of Hoang-Reed that we had hoped was true, and Kierstead showed us a pretty proof of the $k = 3$ case, that might be generalizable); and one on a question of Thomasse on the maximum number of two-edge induced directed paths in an n-vertex digraph. This last was quite fruitful: Thomasse conjectures that the maximum is $n^3/15$; Furedi (on the last day) prove an upper bound of $n^3/12$; and a couple of weeks later, Bondy improved this to $2n^3/25$, using his technique of subgraph counting. There was one other "secret" discussion group: Charbit, Devos and Thomasse had an new idea that they wanted to keep private for a while, because they thought it might work. They were thinking about a result in graph theory that they thought could be proved more simply and in a more general form if described as a problem about digraphs with no transitive triangles.

In Thomasse's talk, he described a beautiful new conjecture of Lichiardopol which would generalize several other conjectures. But a couple of weeks after the workshop, Mader found a counter-example to Lichiardopol's conjecture.